SO‘Z BOSHI

Ushbu o‘quv-uslubiy materiallar qurilish ta’lim yo‘nalishlarida sirtdan o‘qiyotgan talabalar uchun mo‘ljallangan va «Oliy matematika» fanini o‘rganishda ular uchun ko‘rsatma vazifasini o‘taydi. U sirtqi talabalar uchun nazorat ishlarini bajarishlariga oid asosiy tavsiyalarni va shuningdek fanning «Chiziqli algebra elementlari», «Vektorli va analitik geometriya» va «Matematik analizga kirish» bo‘limlarini o‘rganish bo‘yicha uslubiy ko‘rsatmalarni o‘z ichiga oladi.

Uslubiy ko‘rsatmada «Oliy matematika» fanidan savollar, tavsiya qilinayotgan adabiyotlar ro‘yxati va nazorat ishlari uchun yigirma besh variantdan iborat topshiriiqlar keltirilgan. Nazorat ishlarining har bir topshirig‘iga oid namunaviy misol-masalalar yechib ko‘rsatilgan.

Materiallarda nazorat topshiriqlari yigirma besh variant uchun berilgan bo‘lib ular uchta qismga ajratilgan.

Ushbu qo‘llanma «Matematika va tabiiy fanlar» kafedrasi tomonidan qurilish ta’lim yo‘nalashlari sirtqi talabalarini o‘quv-uslubiy ta’minlashning tarkibiy qismlaridan biri hisoblanadi.

SIRTDAN O‘QIYOTGAN TALABALAR USHUN NAZORAT ISHLARINI BAJARISH BO‘YICHA UMUMIY TAVSIYALAR

1. Sirtqi talaba fanni o‘rganish jarayonida oily matematikaning turli bo‘limlaridan nazorar ishlaruini bajarishi lozim. Bu nazorat ishlari o‘qituvchi tomonidan taqriz qilinadi. Bajarilgan ishga yozilgan taqriz talabaga uning materialni o‘zlashtirganligi bo‘yicha baho berish imkonini beradi, mavjud kamchiliklarini ko‘rsatadi va keyingi ishlarini muvofiqlashtiradi va o‘qituvchining qo‘yiladigan savollarni tizimlashtirishida yordam beradi.

2. O‘rganilayotgan material bo‘yicha yetarli sondagi misol va masala yechmasdan talaba nazorat ishini bajarishga kirishmasligi lozim.

3. Har bir nazorat ishi mustaqil bajarilishi kerak. Mustaqil bajarilmagan nazorat ishi taqrizchi - o‘qituvchiga uning ishida materialni o‘zlashtirish bo‘yicha kamchiliklarni ko‘rsatishi uchun imkon bermaydi, natijada talaba kerakli bilimga ega bo‘lmasdan yakuniy nazoratni topshirish uchun tayyor bo‘lmasligi mumkin.

4. Nazorat ishi o‘z vaqtida topshirilishi lozim. Bu talabning bajarilmasligi taqrizchi - o‘qituvchiga talabaning kamchiliklarini o‘z vaqtida ko‘rsatish imkonini bermaydi va ishning taqriz qilinishi vaqtini cho‘zilishiga olib keladi.

5. Nazarat ishini bajarish va rasmiylashlashtirishda talaba quyidagi qoidalarga qat’iy amal qilishi lozim:

a) nazorat ishi alohida daftarga taqrizchi - o‘qituvchining qaydlari uchun xoshiya qoldirilgan holda bajarilshi kerak;

b) daftarning muqavasida quyidagilar qayd etilishi lozom:

- oily matematikadan nazaorat ishi va uning tartib raqami;

- talabaniing familiyasi va ismi-sharifi, reyting daftarchasining nomeri;
- fakultet, kurs, guruh;
- ishning oily o‘quv yurtiga jo‘natilgan sanasi va talabaning manzili.

v) masalalarning yechimi uning keltirilgan tartibida joylashtirilishi kerak;

g) har bir masalani yechishdan oldin uning sharti zarur joylarda harfli ifodalar o‘zining variantiga mos qiymatlar bilan almashtirilgan holda to‘liq ko‘chirilishi kerak;

d) masala yechimining asosiy bosqichlari qisqa va lo‘nda izohlar bilan berilishi lozim;

e) nazorat ishining oxirida foydalanilgan adabiyotlar ro‘yxati berilishi kerak.

6. Talaba reyting daftarchasi nomerining oxirgi ikki raqamiga mos variantni bajaradi. Bunda bu ikki raqam 25 ga bo‘linadi va qoldiq talaba bajarishi kerak bo‘lgan variant nomerini bildiradi. Agar bu ikki raqam 00, 25, 50, 75 dan iborat bo‘lsa, talaba 25- variantni bajaradi.

7. Taqriz qilingan ishni olgandan so‘ng talaba taqrizchi tomonidan ko‘rsatilgan kamchiliklarni tuzatishi va ishni qayta taqrizga jo‘natishi lozim.

8. Belgilangan tartibda taqrizdan o‘tgan va inobatga olingan (zachet qilingan) nazorat ishlarini topshirmagan talaba yakuniy nazoratga kiritilmaydi.

«OLIY MATEMATIKA» KURSIDAN SAVOLLAR RO‘YXATI.

Chiziqli algebra elementlari

Ikkinchi va uchinchi tartibli determinantlar. Determinantning xossalari. n-tartibli determinantlarni hisoblash.

Matritsa va uning turlari. Matritsalar ustida arifmetik amallar. Teskari matritsa. Matritsaning rangi.

Chiziqli algebraik tenglamalar sistemasi. Chiziqli tenglamalar sistemasini yechishning Gauss usuli. n noma’lumli m ta chiziqli tenglamalar sistemasini tekshirish va yechish. Xosmas tenglamalar sistemasini yechish. Bir jinsli chiziqli tenglamalar sistemasi. Chiziqli tenglamalar sistemasini matematik paketlarda yechish.

Vektorli algebra elementlari va analitik geometriya

Vektorlar ustida chiziqli amallar. Vektorning o‘qdagi proeksyasi. Vektorlarning chiziqli bog‘liqligi, bazis. Dekart koordinatalar sistemasida vektorlar.

Ikki vektorning skalyar ko‘paytmasi. Ikki vektorning vektor ko‘paytmasi. Uchta vektorning aralash ko‘paytmasi.

Tekislikdagi chiziq. Tekislikdagi to‘g‘ri chiziq tenglamalari. Tekislikda ikki to‘g‘ri chiziqning o’zaro joylashishi. Nuqtadan to‘g‘ri chiziqgacha bo‘lgan masofa.

Ikkinchi tartibli chiziqlarning umumiy tenglamasi. Aylana va ellips. Giperbola. Parabola.

Qutb koordinatalari. Qutb koordinatalar sistemasiida chiziqlar

Fazoda sirt va chiziq. Tekislik tenglamalari. Fazoda ikki tekislikning o’zaro joylashishi. Nuqtadan tekislikkacha bo‘lgan masofa.

Fazodagi to‘g‘ri chiziqning tenglamalari. Fazoda ikki to‘g‘ri chiziqning o’zaro joylashishi. Fazoda to‘g‘ri chiziq bilan tekislikning o’zaro joylashishi. Nuqtadan fazodagi to‘g‘ri chiziqgacha bo‘lgan masofa.

Ikkinchi tartibli sirtlarning umumiy tenglamalari. Sfera va ellipsoidlar. Giperboloidlar. Konus sirtlar. Paraboloidlar. Silindrik sirtlar. Ikkinchi tartibli sirtlarning to’g’ri chiziqli yasovchilari.

Matematik analizga kirish

To‘plam. Sonli to’plamlar. Matematik mantiq elementlari. Haqiqiy sonlar.

Kompleks son tushunchasi va tasviri. Kompleks sonlarning yozilish shakllari. Kompleks sonlar ustuda amallar

Sonli ketma-ketliklar. Cheksiz katta va cheksiz kichik ketma-ketliklar. Ketma-ketlikning limiti. Yaqinlashuvchi ketma-ketliklar. e soni.

Bir o‘zgaruvchining funksiyasi. Asosiy elementar funksiyalar. Teskari funksiya. Murakkab funksiya. Elementar funksiyalar sinfi. Giperbolik funksiyalar. Oshkormas va parametrik ko’rinishda berilgan funksiyalar.

Funksiyaning limiti. Cheksiz kichik funksiyalar.

Funksiya uzluksizligining ta’riflari. Uzluksiz funksiyalarning xossalari. Funksiyaning uzilish nuqtalari. Tekis uzluksizlik.

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NAZORAT ISHINI BAJARISH BO‘YICHA USLUBIY KO‘RSATMALAR

Uslubiy ko‘rsatmaning ushbu bandida nazorat ishlarining namunaviy masalalari yechib ko‘rsatilgan. Masalalarning yechimi talaba nazorat ishini bajarishi jarayonida o‘rganishi kerak bo‘lgan mavzular bo‘yicha keltirilgan. Masalalarning yechimi talaba o‘zining variantini bajarishida faodalanilishi mumkin bo‘lgan formula va tushunchalarni o‘z ichiga olgan. Ta’kidlash joizki, bu formula va nazariy tushunchalar faqat amaliy mashg‘ulotlarda va nazorat ishlarini bajarilishida qo‘llanilishi mumkin. Ular yakuniy nazoratni topshirish uchun yetarli emas.

1- MAVZU. CHIZIQLI ALGEBRA ELEMENTLARI

1-masala. 1. Berilgan determinantni hisoblang: a) $i-$satr elementlari bo‘yicha yoyib; b) $j-$ ustun elementlari bo‘yicha yoyib; c) $j-$ ustundagi bittadan boshqa elementlarni nolga aylantirib va shu ustun elementlari bo‘yicha yoyib.

$$\begin{vmatrix} -4 & 1 & 2 & 0 \\ 2 & -1 & 2 & 3 \\ -3 & 0 & 1 & 1 \\ 2 & 1 & 2 & 3 \end{vmatrix}, i=2,\ j=2.$$

Yechish. a) Determinantni $i=2-$satr elementlari bo‘yicha yoyamiz.

$$\Delta = a_{21}A_{21} + a_{22}A_{22} + a_{23}A_{23} + a_{24}A_{24} = -a_{21}M_{21} + a_{22}M_{22} - a_{23}M_{23} + a_{24}A_{24} = .$$

$$= -2\cdot\begin{vmatrix} 1 & 2 & 0 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{vmatrix} - 1\cdot\begin{vmatrix} -4 & 2 & 0 \\ -3 & 1 & 1 \\ 2 & 2 & 3 \end{vmatrix} - 2\cdot\begin{vmatrix} -4 & 1 & 0 \\ -3 & 0 & 1 \\ 2 & 1 & 3 \end{vmatrix} + 3\cdot\begin{vmatrix} -4 & 1 & 2 \\ -3 & 0 & 1 \\ 2 & 1 & 2 \end{vmatrix} =$$

$$= -2\cdot(3+2+0-0-2-0) - (-12+4+0-0+8+18) - 2\cdot(0+2+0-0+4+9) +$$

$$+ 3(0+2-6-0+4+6) = -6-18-30+18 = -36.$$

b)Determinantni $j=2-$ustun elementlari bo‘yicha yoyamiz:

$$\Delta = a_{12}A_{12} + a_{22}A_{22} + a_{32}A_{32} + a_{42}A_{42} = -a_{12}M_{12} + a_{22}M_{22} - a_{32}M_{32} + a_{42}A_{42} =$$

$$= -1\cdot\begin{vmatrix} 2 & 2 & 3 \\ -3 & 1 & 1 \\ 2 & 2 & 3 \end{vmatrix} - 1\cdot\begin{vmatrix} -4 & 2 & 0 \\ -3 & 1 & 1 \\ 2 & 2 & 3 \end{vmatrix} - 0 + 1\cdot\begin{vmatrix} -4 & 2 & 0 \\ 2 & 2 & 3 \\ -3 & 1 & 1 \end{vmatrix} =$$

$$= -(6+4-18-6-4+18) - (-12+4+0-0+8+18) +$$

$$+ (-8-18+0-0+12-4) = -0-18-18 = -36.$$

c) Determinantni $j=2-$ustundagi bittadan boshqa elementlarni nolga aylantirib va shu ustun elementlari bo‘yicha yoyib hisoblaymiz.

Buning uchun:

– 1-satr elementlarini 2- satrning mos elementlariga qo‘shamiz;

– 1-satr elementlarini (-1) ga ko‘paytirib 4-satrning mos elementlariga qo‘shamiz;

– determinantni 2-ustun elementlari bo‘yicha yoyamiz

$$\Delta=\begin{vmatrix}-4 & 1 & 2 & 0\\ -2 & 0 & 4 & 3\\ -3 & 0 & 1 & 1\\ 6 & 0 & 0 & 3\end{vmatrix}=1\cdot(-1)^{1+2}\cdot\begin{vmatrix}-2 & 4 & 3\\ -3 & 1 & 1\\ 6 & 0 & 3\end{vmatrix}=-\begin{vmatrix}-2 & 4 & 3\\ -3 & 1 & 1\\ 6 & 0 & 3\end{vmatrix}.$$

Uchinchi tartibli determinantda $2-$ustunning $2-$satri elementidan boshqa elementlarini nolga aylantiramiz. Bunda a_{32} element nolga teng bo‘lgani uchun faqat a_{12} elementni nolga aylantiramiz. Buning uchun 1-satrga (-4) ga ko‘paytirilgan 2-satrni qo‘shamiz, hosil bo‘lgan determinantni $2-$ustun elementlari bo‘yicha yoyamiz va kelib chiqqan ikkinchi tartibli determinantni hisoblaymiz:

$$\Delta=-\begin{vmatrix}10 & 0 & -1\\ -3 & 1 & 1\\ 6 & 0 & 3\end{vmatrix}=-1\cdot(-1)^{2+2}\cdot\begin{vmatrix}10 & -1\\ 6 & 3\end{vmatrix}=-36.$$

2-masala. A, B matritsalar va α, β sonlari berilgan. $\alpha A+\beta B$, AB, A^{-1} matritsalarni toping.

$$A=\begin{pmatrix}4 & 1 & -4\\ 2 & -4 & 6\\ 1 & 2 & -1\end{pmatrix},\qquad B=\begin{pmatrix}0 & -1 & 1\\ 2 & 5 & 0\\ 1 & 1 & 2\end{pmatrix},\qquad \alpha=-4,\quad \beta=4.$$

Yechish. a) $\alpha A+\beta B$ matritsani topish uchun A matritsa elementlarini α ga, B matritsa elementlarini β ga ko‘paytiramiz va hosil qilingan αA va βB matritsalarning mos elementlarini qo‘shamiz:

$$\alpha A+\beta B=(-4)\cdot\begin{pmatrix}4 & 1 & -4\\ 2 & -4 & 6\\ 1 & 2 & -1\end{pmatrix}+4\cdot\begin{pmatrix}0 & -1 & 1\\ 2 & 5 & 0\\ 1 & 1 & 2\end{pmatrix}=$$

$$=\begin{pmatrix}-16 & -4 & 16\\ -8 & 16 & -24\\ -4 & -8 & 4\end{pmatrix}+\begin{pmatrix}0 & -4 & 4\\ 8 & 20 & 0\\ 4 & 4 & 8\end{pmatrix}=$$

$$=\begin{pmatrix} -16+0 & -4+(-4) & 16+4 \\ -8+8 & 16+20 & -24+0 \\ -4+4 & -8+4 & 4+8 \end{pmatrix}=\begin{pmatrix} -16 & -8 & 20 \\ 0 & 36 & -24 \\ 0 & -4 & 12 \end{pmatrix}.$$

b) AB martitsani matritsalarni ko‘paytirish qoidasi asosida topamiz:

$$AB=\begin{pmatrix} 4 & 1 & -4 \\ 2 & -4 & 6 \\ 1 & 2 & -1 \end{pmatrix}\cdot\begin{pmatrix} 0 & -1 & 1 \\ 2 & 5 & 0 \\ 1 & 1 & 2 \end{pmatrix}=$$

$$=\begin{pmatrix} 0+2-4 & -4+5-4 & 4+0-8 \\ 0-8+6 & -2-20+6 & 2+0+12 \\ 0+4-1 & -1+10-1 & 1+0-2 \end{pmatrix}=\begin{pmatrix} -2 & -3 & -4 \\ -2 & -16 & 14 \\ 3 & 8 & -1 \end{pmatrix}.$$

c) A matritsa determinantini hisoblaymiz:

$$|A|=\begin{vmatrix} 4 & 1 & -4 \\ 2 & -4 & 6 \\ 1 & 2 & -1 \end{vmatrix}=16+6-16-16-48+2=-56\neq 0.$$

A_{ij} algebraik to‘ldiruvchilarni topamiz:

$$A_{11}=\begin{vmatrix} -4 & 6 \\ 2 & -1 \end{vmatrix}=-8,\qquad A_{12}=-\begin{vmatrix} 2 & 6 \\ 1 & -1 \end{vmatrix}=8,\qquad A_{13}=\begin{vmatrix} 2 & -4 \\ 1 & 2 \end{vmatrix}=8,$$

$$A_{21}=-\begin{vmatrix} 1 & -4 \\ 2 & -1 \end{vmatrix}=-7,\qquad A_{22}=\begin{vmatrix} 4 & -4 \\ 1 & -1 \end{vmatrix}=0,\qquad A_{23}=-\begin{vmatrix} 4 & 1 \\ 1 & 2 \end{vmatrix}=-7,$$

$$A_{31}=\begin{vmatrix} 1 & -4 \\ -4 & 6 \end{vmatrix}=-10,\qquad A_{32}=-\begin{vmatrix} 4 & -4 \\ 2 & 6 \end{vmatrix}=-32,\qquad A_{33}=\begin{vmatrix} 4 & 1 \\ 2 & -4 \end{vmatrix}=-18.$$

Bundan

$$A^{-1}=\frac{1}{|A|}\begin{pmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{pmatrix}=\frac{1}{-56}\begin{pmatrix} -8 & -7 & -10 \\ 8 & 0 & -32 \\ 8 & -7 & -18 \end{pmatrix}=\begin{pmatrix} \frac{1}{7} & \frac{1}{8} & \frac{5}{28} \\ -\frac{1}{7} & 0 & \frac{16}{28} \\ -\frac{1}{7} & \frac{1}{8} & \frac{9}{28} \end{pmatrix}.$$

3-masala. Tenglamalar sistemalarini birgalikda bo‘lish-bo‘lmasligini Kroniker-Kapelli teoremasi bilan tekshiring. Birgalikda bo‘lgan sistemani Kramer formulalari orqali, matritsalar va Gauss usullari bilan yeching.

$$\begin{cases} 2x_1+x_2+3x_3=-3, \\ x_1-5x_2-x_3=-10, \\ 3x_1+4x_2+x_3=4. \end{cases}$$

Yechish. Sistemaning kengaytirilgan matritsasi ustida elementar almashtirishlar bajaramiz:

$$C=\left(\begin{array}{ccc|c} 2 & 1 & 3 & -3\\ 1 & -5 & -1 & -10\\ 3 & 4 & 1 & 4\end{array}\right)\sim\left(\begin{array}{ccc|c} 1 & -5 & -1 & -10\\ 2 & 1 & 3 & -3\\ 3 & 4 & 1 & 4\end{array}\right)\begin{array}{c}\\ -2\\ -3\end{array}\sim$$

$$\sim\left(\begin{array}{ccc|c} 1 & -5 & -1 & -10\\ 0 & 11 & 5 & 17\\ 0 & 19 & 4 & 34\end{array}\right)\sim:11\left(\begin{array}{ccc|c} 1 & -5 & -1 & -10\\ 0 & 1 & \frac{5}{11} & \frac{17}{11}\\ 0 & 19 & 4 & 34\end{array}\right)\sim$$

$$\sim -19\left(\begin{array}{ccc|c} 1 & -5 & -1 & -10\\ 0 & 1 & \frac{5}{11} & \frac{17}{11}\\ 0 & 0 & -\frac{51}{11} & \frac{51}{11}\end{array}\right)\sim:\left(-\frac{51}{11}\right)\left(\begin{array}{ccc|c} 1 & -5 & -1 & -10\\ 0 & 1 & \frac{5}{11} & \frac{17}{11}\\ 0 & 0 & 1 & -1\end{array}\right).$$

$r(A)=3=3=r(C)$

Demak, sistema Kroniker-Kapelli teoremasiga ko‘ra aniq sistema.

1) *Sistemani Kramer formulalari bilan yechamiz.*

Sistemaning determinantini va yordamchi determinantlarni hisoblaymiz:

$$\Delta=\begin{vmatrix} 2 & 1 & 3\\ 1 & -5 & -1\\ 3 & 4 & 1\end{vmatrix}=51;\qquad \Delta x_1=\begin{vmatrix} -3 & 1 & 3\\ -10 & -5 & -1\\ 4 & 4 & 1\end{vmatrix}=-51;$$

$$\Delta x_2=\begin{vmatrix} 2 & -3 & 3\\ 1 & -10 & -1\\ 3 & 4 & 1\end{vmatrix}=102;\quad \Delta x_3=\begin{vmatrix} 2 & 1 & -3\\ 1 & -5 & -10\\ 3 & 4 & 4\end{vmatrix}=-51;$$

Tenglamaning yechimini Kramer formulalari bilan topamiz:

$$x_1=\frac{\Delta x_1}{\Delta}=\frac{-51}{51}=-1;\quad x_2=\frac{\Delta x_2}{\Delta}=\frac{102}{51}=2;$$

$$x_3=\frac{\Delta x_3}{\Delta}=\frac{-51}{51}=-1.$$

2) *Sistemani matritsalar usuli bilan yechamiz.*

Sistema uchun $\Delta=51$.

Sistema determinantining algebraik to‘ldiruvchilarini topamiz:

$$A_{11}=\begin{vmatrix}-5 & -1\\ 4 & 1\end{vmatrix}=-1; \qquad A_{12}=-\begin{vmatrix}1 & -1\\ 3 & 1\end{vmatrix}=-4; \qquad A_{13}=\begin{vmatrix}1 & -5\\ 3 & 4\end{vmatrix}=19;$$

$$A_{21}=-\begin{vmatrix}1 & 3\\ 4 & 1\end{vmatrix}=11; \qquad A_{22}=\begin{vmatrix}2 & 3\\ 3 & 1\end{vmatrix}=-7; \qquad A_{23}=-\begin{vmatrix}2 & 1\\ 3 & 4\end{vmatrix}=-5;$$

$$A_{31}=\begin{vmatrix}1 & 3\\ -5 & -1\end{vmatrix}=14; \qquad A_{32}=-\begin{vmatrix}2 & 3\\ 1 & -1\end{vmatrix}=5; \qquad A_{33}=\begin{vmatrix}2 & 1\\ 1 & -5\end{vmatrix}=-11.$$

U holda

$$A^{-1}=\frac{1}{51}\begin{pmatrix}-1 & 11 & 14\\ -4 & -7 & 5\\ 19 & -5 & -11\end{pmatrix}.$$

Tenglamaning yechimini $X=A^{-1}B$ formula bilan topamiz:

$$X=A^{-1}B=\frac{1}{51}\begin{pmatrix}-1 & 11 & 14\\ -4 & -7 & 5\\ 19 & -5 & -11\end{pmatrix}\cdot\begin{pmatrix}-3\\ -10\\ 4\end{pmatrix}=\frac{1}{51}\begin{pmatrix}3-110+56\\ 12+\ 70+20\\ -57+\ 50-44\end{pmatrix}=\frac{1}{51}\begin{pmatrix}-51\\ 102\\ -51\end{pmatrix}=\begin{pmatrix}-1\\ 2\\ -1\end{pmatrix}.$$

Demak, $x_1=-1$, $x_2=2$, $x_3=-1$.

3) *Sistemani Gauss usuli bilan yechamiz.*

Gauss usulining 1-bosqichi yuqorida sistemani tekshirishda uning kengaytirilgan matritsasida bajarildi va quyidagi ko‘rinish hosil qilindi:

$$\left(\begin{array}{ccc|c}1 & -5 & -1 & -10\\ 0 & 1 & \frac{5}{11} & \frac{17}{11}\\ 0 & 0 & 1 & -1\end{array}\right).$$

Gauss usulining 2-bosqichini bajaramiz:

$$\begin{cases}x_1-5x_2-\ \ x_3=-10,\\ \qquad x_2+\frac{5}{11}x_3=\frac{17}{11},\\ \qquad\qquad x_3=-1\end{cases}\Rightarrow\begin{cases}\qquad\qquad x_3=-1,\\ \quad x_2+\frac{5}{11}\cdot(-1)=\frac{17}{11},\\ x_1-5x_2-\ (-1)=-10\end{cases}\Rightarrow$$

$$\begin{cases}\qquad x_3=-1,\\ \qquad x_2=\ 2,\\ x_1-5\cdot 2_2=-11\end{cases}\Rightarrow\begin{cases}x_1=-1,\\ x_2=\ 2,\\ x_3=-1.\end{cases}$$

4-masala. Bir jinsli tenglamalar sistemasini yeching.

$$\begin{cases} 5x_1 - x_2 - x_3 = 0, \\ x_1 + 3x_2 + 7x_3 = 0, \\ 3x_1 + x_2 + 3x_3 = 0. \end{cases}$$

Yechish a) Sistema matritsasi ustida elementar almashtirishlar bajaramiz:

$$A = \begin{array}{c} -5 \\ \\ -3 \end{array} \begin{pmatrix} 5 & -1 & -1 \\ 1 & 3 & 7 \\ 3 & 1 & 3 \end{pmatrix} \sim \begin{array}{c} \\ \\ -2 \end{array} \begin{pmatrix} 0 & -16 & -36 \\ 1 & 3 & 7 \\ 0 & -8 & -18 \end{pmatrix} \sim \begin{pmatrix} 0 & -16 & -36 \\ 1 & 3 & 7 \\ 0 & 0 & 0 \end{pmatrix}.$$

$r(A) = 2,\ n = 3,\ r < n.$ Demak, sistema cheksiz ko'p yechimga ega.

Ularni topamiz:

$$\begin{cases} 5x_1 - x_2 - x_3 = 0, \\ x_1 + 3x_2 + 7x_3 = 0 \end{cases} \Rightarrow \begin{cases} 5x_1 - x_2 = x_3, \\ x_1 + 3x_2 = -7x_3. \end{cases}$$

$$\Delta = \begin{vmatrix} 5 & -1 \\ 1 & 3 \end{vmatrix} = 16, \quad \Delta x_1 = \begin{vmatrix} x_3 & -1 \\ -7x_3 & 3 \end{vmatrix} = -4x_3, \quad \Delta x_2 = \begin{vmatrix} 5 & x_3 \\ 1 & -7x_3 \end{vmatrix} = -36x_3.$$

$$x_1 = \frac{\Delta x_1}{\Delta} = -\frac{x_3}{4},\ x_2 = \frac{\Delta x_2}{\Delta} = -\frac{9x_3}{4}.$$

Erkin noma'lumni $x_3 = -4k$ ($k -$ixtiyoriy son) deb, sistemaning umumiy yechimini topamiz: $x_1 = k,\ x_2 = 9k,\ x_3 = -4k.$

2- MAVZU. VEKTORLI ALGEBRA ELEMENTLARI VA ANALITIK GEOMETRIYA

5-masala. ABC uchburchak uchlarining koordinatalari berilgan: a) C uchdan tushirilgan balandlik tenglamasini tuzing va uning uzunligini toping; b) B uchdan o'tkazilgan mediana tenglamasini tuzing va uchburchak medianalarining kesishish nuqtalarini toping; c) A burchakning radian qiymatini hisoblang va uning bissektrisasi tenglamasini tuzing.

$A(0-2),\ B(-5;10),\ C(4;1).$

Yechish. a) AB tomon tenglamasini berilgan ikki nuqtadan o'tuvchi to'g'ri chiziq tenglamasi formulasidan topamiz:

$$\frac{x+5}{0+5} = \frac{y-10}{-2-10},\quad 12x + 5y + 10 = 0\ (AB).$$

Bundan

$$y = -\frac{12}{5}x - 2,\ k_1 = -\frac{12}{5}.$$

CM balandlik AB tomonga perpendikular bo‘lib, C nuqtadan o‘tadi (1-shakl). Shu sababli uning tenglamasi

$$y-1=k(x-4),\ \ y-1=-\frac{1}{k_1}(x-4),\ \ y-1=\frac{5}{12}(x-4),$$
$$5x-12y-8=0\ (CM).$$

CM balandlik uzunligi C nuqtadan AB to‘g‘ri chiziqqacha bo‘lgan masofaga teng.

Demak,

$$|CM|=\frac{|12\cdot 4+5\cdot 1+10|}{\sqrt{12^2+5^2}}=\frac{63}{13}(u.b.).$$

b) AC tomon o‘rtasi $N(x;y)$ nuqtada bo‘lsin. U holda kesmaning o‘rtasi koordinatalarini topish formulasiga ko‘ra:

$$x=\frac{0+4}{2}=2,\quad y=\frac{-2+1}{2}=-\frac{1}{2}\ \text{ yoki }\ N\left(2;-\frac{1}{2}\right).$$

BN mediana tenglamasini tuzamiz:

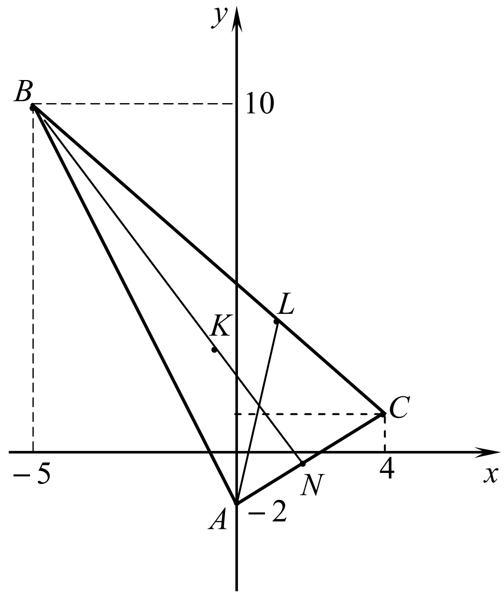
$$\frac{x+5}{2+5}=\frac{y-10}{-\frac{1}{2}-10},\quad 3x+2y-5=0\ (BN).$$

Uchburchak medianalarining xossasiga ko‘ra medianalarning kesishish nuqtasi $K(x;y)$ da $\frac{|BK|}{|KN|}=\frac{2}{1}=2$ bo‘ladi.

U holda

$$x=\frac{-5+2\cdot 2}{1+2}=-\frac{1}{3};\ \ y=\frac{10-2\cdot\frac{1}{2}}{1+2}=3$$

yoki $K\left(-\frac{1}{3};3\right)$.



1-shakl.

c) AC tomon tenglamasini tuzamiz:

$$\frac{x-0}{4-0}=\frac{y+2}{1+2},\quad 3x-4y-8=0\ (AC).$$

AB va AC tomonlar orasida burchak $\angle A=\varphi$ bo‘lsin. Uni ikki to‘g‘ri chiziq orasidagi burchak formulasidan foydalanib hisoblaymiz:

$$\cos\varphi=\frac{12\cdot 3+5\cdot(-4)}{\sqrt{12^2+5^2}\cdot\sqrt{3^2+(-4)^2}}=\frac{16}{65}$$

yoki $\varphi=\arccos\frac{16}{65}\approx 0{,}3134.$

A burchak bissektrisasi CB tomon bilan $L(x;y)$ nuqtada kesishsin (1-shakl).

Uchburchak bissektrisasining xossasiga ko‘ra

$$\frac{|\overrightarrow{CL}|}{|\overrightarrow{LB}|}=\frac{|\overrightarrow{AC}|}{|\overrightarrow{AB}|}.$$

$|\overrightarrow{AC}|=\sqrt{(4-0)^2+(1+2)^2}=5$ va $|\overrightarrow{AB}|=\sqrt{(-5-0)^2+(10+2)^2}=13$ ekanidan

$$\frac{|\overrightarrow{CL}|}{|\overrightarrow{LB}|}=\frac{5}{13}.$$

U holda

$$x=\frac{4+\frac{5}{13}\cdot(-5)}{1+\frac{5}{13}}=\frac{3}{2},\quad y=\frac{1+\frac{5}{13}\cdot 10}{1+\frac{5}{13}}=\frac{7}{2}\quad \text{yoki}\quad L\left(\frac{3}{2};\frac{7}{2}\right).$$

Ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasidan topamiz:

$$\frac{x-0}{\frac{3}{2}-0}=\frac{y+2}{\frac{7}{2}+2}$$

yoki

$$11x-3y-6=0\ (AL).$$

6-masala. 1. Har bir $M(x;y)$ nuqtasidan berilgan $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalargacha bo‘lgan masofalar nisbati a ga teng bo‘lgan chiziq tenglamasini tuzing.

$$A(3;-2),\ B(4;6),\ a=\frac{3}{5}.$$

Yechish. Ikki nuqta orasidagi masofa formulasidan topamiuz:

$$|AM|=\sqrt{(x-3)^2+(y+2)^2},\quad |BM|=\sqrt{(x-4)^2+(y-6)^2}.$$

Misolning shartiga ko‘ra

$$\frac{|AM|}{|BM|}=a\ \text{ yoki }\ \frac{\sqrt{(x-3)^2+(y+2)^2}}{\sqrt{(x-4)^2+(y-6)^2}}=\frac{3}{5}.$$

Bu tenglikda almashtirishlar bajaramiz:

$$25(x^2-6x+9+y^2+4y+4)=9(x^2-8x+16+y^2-12y+36),$$

$$25x^2-150x+25y^2+100y+325=9x^2-72x+9y^2-108y+468,$$

$$16x^2-78x+16y^2+208y=143,$$

$$16\left(x^2-\frac{39}{8}x+y^2+13y\right)=143,$$

$$x^2-2\cdot\frac{39}{16}x+\left(\frac{39}{16}\right)^2+y^2+2\cdot\frac{13}{2}y+\left(\frac{13}{2}\right)^2=\frac{143}{16}+\left(\frac{39}{16}\right)^2+\left(\frac{13}{2}\right)^2,$$

$$\left(x-\frac{39}{16}\right)^2+\left(y+\frac{13}{2}\right)^2=\left(\frac{15\sqrt{65}}{16}\right)^2.$$

Bu tenglama markazi $\left(\frac{39}{16};-\frac{13}{2}\right)$ nuqtada joylashgan va radiusi $\frac{15\sqrt{65}}{16}$ ga teng bo‘lgan aylanani aniqlaydi.

2. Har bir $M(x;y)$ nuqtasidan berilgan $A(x_1;y_1)$ nuqtagacha va $x=b$ to‘g‘ri chiziqqacha bo‘lgan masofalar nisbati m ga teng bo‘lgan chiziq tenglamasini tuzing.

$$A(6;0),\ x=\frac{3}{2},\ m=2.$$

Yechish. Ikki nuqta orasidagi masofa va nuqtadan to‘g‘ri chiziqqacha bo‘lgan masofa formulalari bilan topamiz:

$$|AM|=\sqrt{(x-6)^2+(y-0)^2},\quad |BM|=\left|x-\frac{3}{2}\right|.$$

Misolning shartiga ko‘ra

$$\frac{|AM|}{|BM|}=m \text{ yoki } \frac{\sqrt{(x-6)^2+y^2}}{\left|x-\frac{3}{2}\right|}=2.$$

Bundan

$$(x-6)^2+y^2=4\left(x-\frac{3}{2}\right)^2.$$

Bu tenglikda almashtirishlarni bajaramiz:

$$x^2-12x+36+y^2=4\left(x^2-3x+\frac{9}{4}\right),$$

$$x^2-12x+36+y^2=4x^2-12x+9,$$

$$3x^2-y^2=27,$$

$$\frac{x^2}{9}-\frac{y^2}{27}=1.$$

Bu tenglama fokuslari Ox o‘qida joylashgan va yarim o‘qlari $a=3,\ b=3\sqrt{3}$ ga teng bo‘lgan giperbolani aniqlaydi.

7-masala. $ABCD$ piramidaning uchlari berilgan: a) AB qirra tenglamasini tuzing; b) ABC yoq tenglamasini tuzing; c) D uchdan ABC yoqqa tushirilgan balandlik tenglamasini tuzing va uning uzunligini toping; d) C uchdan o‘tuvchi AB qirraga parallel to‘g‘ri chiziq tenglamasini tuzing; e) D uchdan o‘tuvchi AB qirraga perpendikular tekislik tenglamasini tuzing; f) AD qirra bilan ABC yoq orasidagi burchak sinusini toping; g) ABC va ABD yoqlar orasidagi burchak kosinusini toping.

$$A(2;1;7),\ B(3;3;6),\ C(2;-3;9),\ D(1;2;5).$$

Yechish. a) AB qirra tenglamasini berilgan ikki nuqtadan o‘tuvchi to‘g‘ri chiziq tenglamasidan foydalanib tuzamiz:

$$\frac{x-2}{3-2}=\frac{y-1}{3-1}=\frac{z-7}{6-7} \text{ yoki}$$
$$\frac{x-2}{1}=\frac{y-1}{2}=\frac{z-7}{-1} \ (AB).$$

b) ABC yoq tenglamasini berilgan uchta nuqtadan o‘tuvchi tekislik tenglamasi bilan tuzamiz:

$$\begin{vmatrix} x-2 & y-1 & z-7 \\ 1 & 2 & -1 \\ 0 & -4 & 2 \end{vmatrix}=0.$$

Bundan

$$y+2z-15=0 \ (ABC).$$

c) D uchdan tushirilgan DE balandlik ABC yoqqa perpendikular bo‘ladi. Shu sababli DE to‘g‘ri chiziqning yo‘naltiruvchi vektori $\vec{s}=\{p;q;r\}$ sifatida ABC yoqning normal vektori $\vec{n}_1=\{0;1;2\}$ ni olish mumkin. U holda to‘g‘ri chiziqning kanonik tenglamasi formulasiga ko‘ra

$$\frac{x-1}{0}=\frac{y-2}{1}=\frac{z-5}{2} \ (DE).$$

Nuqtadan tekislikkacha bo‘lgan masofa formulasidan topamiz:

$$|DE|=\frac{|0\cdot 1+1\cdot 2+2\cdot 5-15|}{\sqrt{0^2+1^2+2^2}} \text{ yoki } |DE|=\frac{3\sqrt{5}}{5}(u.b.).$$

d) C uchdan o‘tuvchi CF to‘g‘ri chiziq AB qirraga parallel bo‘gani sababli CF to‘g‘ri chiziq va AB qirraning yo‘naltiruvchi vektori $\vec{s}_1=\vec{s}_2=\{1;2;-1\}$ bo‘ladi. U holda

$$\frac{x-2}{1}=\frac{y+3}{2}=\frac{z-9}{-1} \ (CF).$$

e) D uchdan o‘tuvchi tekislik AB qirraga perpendikular bo‘lgani uchun AB to‘g‘ri chiziqning yo‘naltiruvchi vektori $\vec{s}_1=\{1;2;-1\}$ ni izlanayotgan tekislikning normal vektori $\vec{n}_2=\{A;B;C\}$ deb olish mumkin. Tekislik tenglamasini berilgan nuqtadan o‘tuvchi va berilgan vektorga perpendikular tekislik tenglamasi bilan topamiz:

$$1\cdot(x-1)+2\cdot(y-2)+(-1)\cdot(z-5)=0 \text{ yoki}$$
$$x+2y-z=0.$$

f) AD qirra tenglamasini tuzamiz:

$$\frac{x-2}{-1}=\frac{y-1}{1}=\frac{z-7}{-2} \ (AD).$$

AD qirra bilan ABC yoq orasidagi burchak sinusini to‘g‘ri chiziq bilan tekislik orasidagi burchak formulasidan topamiz:

$$\sin\varphi = \frac{0\cdot(-1)+1\cdot 1+2\cdot(-2)}{\sqrt{0^2+1^2+2^2}\cdot\sqrt{(-1)^2+1^2+(-2)^2}} = \frac{-3}{\sqrt{5}\cdot\sqrt{6}} \approx -0{,}54$$

g) ABD yoq tenglamasini tuzamiz:

$$\begin{vmatrix} x-2 & y-1 & z-7 \\ 1 & 2 & -1 \\ -1 & 1 & -2 \end{vmatrix} = 0$$

yoki

$$x-y-z+6=0\ (ABD).$$

ABC va ABD yoqlar orasidagi burchak kosinusini ikki tekislik orasidagi burchak formulasidan foydalanib topamiz:

$$\cos\psi = \frac{0\cdot 1+1\cdot(-1)+2\cdot(-1)}{\sqrt{0^2+1^2+2^2}\cdot\sqrt{1^2+(-1)^2+(-1)^2}} = \frac{-3}{\sqrt{5}\cdot\sqrt{3}} \approx -0{,}77\,.$$

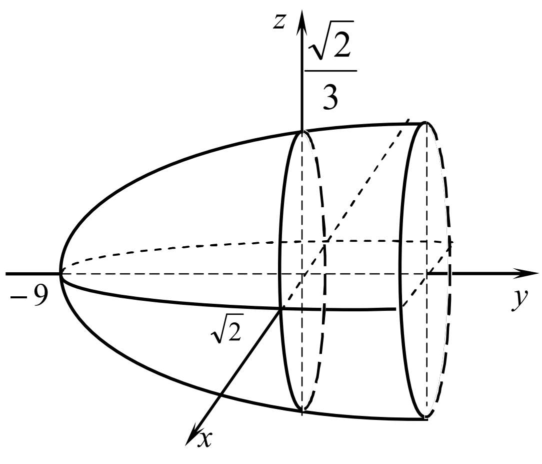
8-masala. Sirt turini aniqlang va shaklini chizing.

a) $9x^2-2y+z^2=18$; b) $4x^2-3y^2=12$.

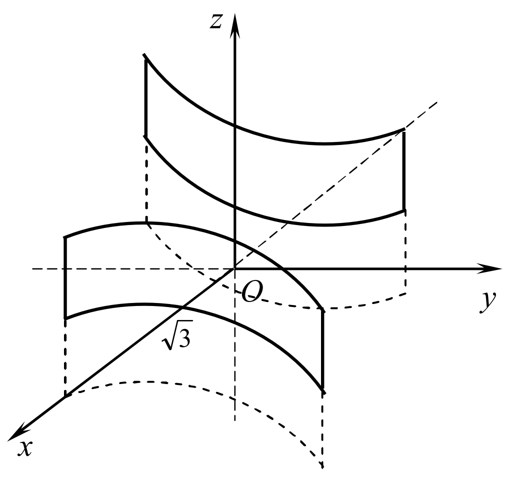
Yechish. a) Sirt tenglamasini kanonik shaklga keltiramiz:

$$9x^2+z^2=2y+18,\quad 9x^2+z^2=2(y+9),\quad \frac{x^2}{\frac{2}{9}}+\frac{z^2}{2}=(y+9).$$

Bu tenglama elliptik paraboloidni aniqlaydi (2-sahkl).



2-shakl.



3-shakl.

b) Berilgan tenglamada $z=0$. Bunda berilgan sirt yasovchilari Oz o‘qqa parallel silindrik sirtdan iborat bo‘ladi.

$4x^2-3y^2=12$ tenglamadan topamiz:

$$\frac{x^2}{3}-\frac{y^2}{4}=1.$$

Bu tenglama giperbola tenglamasi bo‘ladi. Demak, berilgan tenglama giperbolik silindrni aniqlaydi (3-shakl).

3- MAVZU. MATEMATIK ANALIZGA KIRISH

9-masala. Sonli ketma-ketliklarning limitini toping.

a) $x_n=n^2\sqrt{n}-\sqrt{(n^3+1)(n^2-2)}$; b) $x_n=\dfrac{(n+2)!+2(n+1)!}{n!\cdot(1+5+9+...+(4n-3))}$.

Yechish. a) $\lim\limits_{n\to\infty}x_n=\lim\limits_{n\to\infty}(n^2\sqrt{n}-\sqrt{(n^3+1)(n^2-2)})=$

$$=\lim_{n\to\infty}\frac{n^5-n^5+2n^3-n^2+2}{n^2\sqrt{n}+\sqrt{(n^3+1)(n^2-2)}}=\lim_{n\to\infty}\frac{2n^3-n^2+2}{n^2\sqrt{n}+\sqrt{(n^3+1)(n^2-2)}}=$$

$$=\lim_{n\to\infty}\frac{2-\frac{1}{n}+\frac{2}{n^3}}{\sqrt{\frac{1}{n}}+\sqrt{\left(1+\frac{1}{n^3}\right)\left(\frac{1}{n}-\frac{2}{n^3}\right)}}=\frac{2-0+0}{0+\sqrt{(1+0)(0-0)}}=\infty.$$

b) $$x_n=\frac{(n+2)!+2(n+1)!}{n!\cdot(1+5+9+\cdots+(4n-3))}=\frac{n!\cdot(n+1)(n+2+2)}{n!\cdot\left(\frac{1+4n-3}{2}\right)\cdot n}=\frac{(n+1)(n+4)}{n(2n-1)}.$$

Bundan

$$\lim_{n\to\infty}x_n=\lim_{n\to\infty}\frac{(n+1)(n+4)}{n(2n-1)}=\lim_{n\to\infty}\frac{\left(1+\frac{1}{n}\right)\left(1+\frac{4}{n}\right)}{2-\frac{1}{n}}=\frac{(1+0)(1+0)}{2-0}=\frac{1}{2}.$$

10-masala. Limitlarni toping:

a) $\lim\limits_{x\to\infty}\dfrac{x-2x^2+x^4}{3x^4+x^3+1}$; b) $\lim\limits_{x\to-5}\dfrac{x^2-x-30}{x^3+125}$; c) $\lim\limits_{x\to-8}\dfrac{\sqrt{1-x}-3}{\sqrt[3]{x}+2}$;

d) $\lim\limits_{x\to0}\dfrac{x\cdot tg3x}{\cos x-\cos^3x}$; e) $\lim\limits_{x\to-\infty}(x+2)(\ln(2x+3)-\ln(2x-1))$.

Yechish.

a) $$\frac{x-2x^2+x^4}{3x^4+x^3+1}=\frac{x^4\left(\frac{1}{x^3}-\frac{2}{x^2}+1\right)}{x^4\left(3+\frac{1}{x}+\frac{1}{x^4}\right)}=\frac{1-\frac{2}{x^2}+\frac{1}{x^3}}{3+\frac{1}{x}+\frac{1}{x^4}}.$$

U holda

$$\lim_{x\to\infty}\frac{x-2x^2+x^4}{3x^4+x^3+1}=\lim_{x\to\infty}\frac{1-\frac{2}{x^2}+\frac{1}{x^3}}{3+\frac{1}{x}+\frac{1}{x^4}}=\frac{1-\frac{2}{\infty}+\frac{1}{\infty}}{3+\frac{1}{\infty}+\frac{1}{\infty}}=\frac{1-0+0}{3+0+0}=\frac{1}{3}.$$

b) $$\lim_{x\to-5}\frac{x^2-x-30}{x^3+125}=\lim_{x\to-5}\frac{(x+5)(x-6)}{(x+5)(x^2-5x+25)}=\lim_{x\to-5}\frac{x-6}{x^2-5x+25}=-\frac{11}{75}.$$

c) $$\lim_{x\to-8}\frac{\sqrt{1-x}-3}{\sqrt[3]{x}+2}=\lim_{x\to-8}\frac{(\sqrt{1-x}-3)(\sqrt{1-x}+3)}{(\sqrt[3]{x}+2)(\sqrt[3]{x^2}-2\sqrt[3]{x}+4)}\cdot\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+4}{\sqrt{1-x}+3}=$$

$$=\lim_{x\to-8}\frac{-(x+8)}{(x+8)}\cdot\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+4}{\sqrt{1-x}+3}=-\lim_{x\to-8}\frac{\sqrt[3]{x^2}-2\sqrt[3]{x}+4}{\sqrt{1-x}+3}=-\frac{(-2)^2-2\cdot(-2)+4}{3+3}=-2.$$

d) $$\lim_{x\to0}\frac{x\sin3x}{\cos3x\cos x(1-\cos^2x)}=\lim_{x\to0}\frac{x\sin3x}{\cos3x\cos x\sin^2x}=$$

$$=\lim_{x\to0}\frac{1}{\cos3x\cos x}\cdot\lim_{x\to0}\frac{3x^2\cdot\frac{\sin3x}{3x}}{\left(\frac{\sin x}{x}\right)^2\cdot x^2}=1\cdot3\cdot\frac{\lim\limits_{x\to0}\frac{\sin3x}{3x}}{\lim\limits_{x\to0}\left(\frac{\sin x}{x}\right)^2}=3\cdot\frac{1}{1}=3.$$

e) $$\lim_{x\to\infty}(x+2)(\ln(2x+3)-\ln(2x-1))=\lim_{x\to\infty}(x+2)\ln\left(\frac{2x+3}{2x-1}\right)=$$

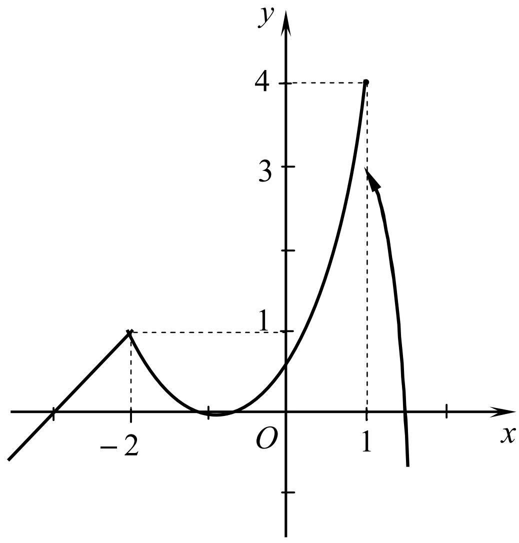
$$=\lim_{x\to\infty}\ln\left(\frac{2x+3}{2x-1}\right)^{x+2}=\lim_{x\to\infty}\ln\left[\left(1+\frac{4}{2x-1}\right)^{\frac{2x-1}{4}}\right]^{\left(\frac{4}{2x-1}\right)(x+2)}=$$

$$\lim_{x\to\infty}\ln e^{\frac{4x+8}{2x-1}}=\lim_{x\to\infty}\frac{4x+8}{2x-1}=2.$$

11-masala. Funksiyani uzluksizlikka tekshiring va grafigini chizing.

$$f(x)=\begin{cases}x+3, & -\infty<x\le-2,\\ (x+1)^2, & -2<x\le1,\\ 4-x^3, & 1<x<+\infty.\end{cases}$$

Yechish. Funksiya $x\in(-\infty;+\infty)$ da aniqlangan. $(-\infty;-2)$, $(-2;1)$, $(1;+\infty)$ oraliqlarda funksiya uzluksiz. $x=-2$, $x=1$ nuqtalarda funksiya analitik berilishni o‘zgartiradi. Shu sababli, bu nuqtalarda funksiya uzilishga ega bo‘lishi mumkin.



4-shakl.

$x=-2$ nuqtada: $f(-2-0)=\lim\limits_{x\to-2-0}(x+3)=1,\ f(-2+0)=\lim\limits_{x\to-2+0}(x+1)^2=1,$ $f(-2)=-2+3=1$. Bundan $f(-2-0)=f(-2+0)=f(-2)$.

Demak, $x=-2$ nuqtada funksiya uzluksiz.

$x=1$ nuqtada:

$f(1-0)=\lim\limits_{x\to1-0}(x+1)^2=4=A_1,\quad f(1+0)=\lim\limits_{x\to1+0}(4-x^3)=3=A_2.$

Demak, $x=1$ sakrash nuqtasi va bu nuqtada funksiya birinchi tur uzilishga ega. Funksiyaning sakrashi $\mu=\left|A_2-A_1\right|=|3-4|=1$ (4-shakl).

12-masala. Funksiyani berilgan nuqtalarda uzluksizlikka tekshiring

$$f(x)=5^{\frac{3}{x+4}};\quad x_1=-4,\quad x_2=-3.$$

Yechish. $x_1=-4$ nuqtada:

$$f(-4-0)=\lim_{x\to-4-0}5^{\frac{3}{x+4}}=0,\quad f(-4+0)=\lim_{x\to-4+0}5^{\frac{3}{x+4}}=+\infty.$$

Demak, $x_1=4$ nuqtada funksiya ikkinchi tur uzilishga ega.

$x_2=-3$ nuqtada:

$$f(-3-0)=\lim_{x\to-3-0}5^{\frac{3}{x+4}}=125,\ f(-3+0)=\lim_{x\to-3+0}5^{\frac{3}{x+4}}=125,$$

$$f(-3)=5^{\frac{3}{-3+4}}=125.$$

Demak, $x_2=-3$ nuqtada funksiya uzluksiz.

NAZORAT ISHINI BAJARISH UCHUN TOPSHIRIQLAR

1-masala. 1. Berilgan determinantni hisoblang: a) $i-$satr elementlari bo‘yicha yoyib; b) $j-$ ustun elementlari bo‘yicha yoyib; c) $j-$ ustundagi bittadan boshqa elementlarni nolga aylantirib va shu ustun elementlari bo‘yicha yoyib.

1.1. $\begin{vmatrix} 1 & 2 & 3 & 4 \\ -2 & 1 & -4 & 3 \\ 3 & 4 & -1 & 2 \\ 4 & 3 & -2 & 1 \end{vmatrix}, i=1, j=2.$ **1.2.** $\begin{vmatrix} -1 & 1 & -2 & 3 \\ 1 & 2 & 2 & 3 \\ -2 & 3 & 1 & 0 \\ 2 & 3 & -2 & 0 \end{vmatrix}, \ i=3,\ j=2.$

1.3. $\begin{vmatrix} 2 & -2 & 0 & 3 \\ 3 & 2 & 1 & -1 \\ 1 & 1 & -2 & 1 \\ 3 & 4 & -4 & 0 \end{vmatrix}, \ i=3,\ \ j=4.$ **1.4.** $\begin{vmatrix} 6 & 0 & -1 & 1 \\ 2 & -2 & 0 & 1 \\ 1 & 1 & -3 & 3 \\ 4 & 1 & -1 & 2 \end{vmatrix}, i=2,\ j=2.$

1.5. $\begin{vmatrix} 1 & -1 & 0 & 3 \\ 3 & 2 & 1 & 1 \\ 1 & 2 & -1 & 3 \\ 4 & 0 & 1 & 2 \end{vmatrix}$, $i=3,\quad j=1.$ **1.6.** $\begin{vmatrix} 5 & 0 & -4 & 2 \\ 1 & -1 & 2 & 1 \\ 4 & 1 & 2 & 0 \\ 1 & 1 & -1 & 1 \end{vmatrix}$, $i=2,\quad j=4.$

1.8. $\begin{vmatrix} 1 & 8 & 2 & -3 \\ 3 & -2 & 0 & 4 \\ 5 & -3 & 7 & -1 \\ 3 & 2 & 0 & 2 \end{vmatrix}$, $i=1,\quad j=4.$ **1.9.** $\begin{vmatrix} 2 & -3 & 4 & 1 \\ 4 & -2 & -3 & 2 \\ 3 & 0 & 2 & 1 \\ 3 & -1 & -4 & 3 \end{vmatrix}$, $i=2,\quad j=4.$

1.10. $\begin{vmatrix} 0 & 4 & 1 & 1 \\ -4 & 2 & 1 & 3 \\ 0 & 1 & 2 & -2 \\ 1 & 3 & 4 & -3 \end{vmatrix}$, $i=4,\quad j=3.$ **1.11.** $\begin{vmatrix} 0 & -2 & 1 & 7 \\ 4 & -8 & 2 & -3 \\ 10 & 1 & -5 & 4 \\ -8 & 3 & 2 & -1 \end{vmatrix}$, $i=4,\quad j=2.$

1.12. $\begin{vmatrix} 5 & -3 & 7 & -1 \\ 3 & 2 & 0 & 2 \\ 2 & 1 & 4 & -6 \\ 3 & -2 & 9 & -4 \end{vmatrix}$, $i=3,\quad j=4.$ **1.13.** $\begin{vmatrix} 4 & -1 & 1 & 5 \\ 0 & 2 & -2 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 1 & 2 \end{vmatrix}$, $i=1,\quad j=2.$

1.14. $\begin{vmatrix} 2 & 1 & 2 & 0 \\ 3 & 4 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 1 & 2 & -3 & -2 \end{vmatrix}$, $i=2,\quad j=3.$ **1.15.** $\begin{vmatrix} 3 & 2 & 0 & -2 \\ 1 & -1 & 2 & 3 \\ 4 & 5 & 1 & 0 \\ -1 & 2 & 3 & -3 \end{vmatrix}$, $i=3,\quad j=1.$

1.16. $\begin{vmatrix} 3 & 1 & 2 & -3 \\ 4 & -1 & 2 & 4 \\ 1 & -1 & 1 & 1 \\ 4 & -1 & 2 & 5 \end{vmatrix}$, $i=1,\quad j=3.$ **1.17.** $\begin{vmatrix} 3 & 1 & 2 & 0 \\ 5 & 0 & -6 & 1 \\ -2 & 2 & 1 & 3 \\ -1 & 3 & 2 & 1 \end{vmatrix}$, $i=3,\quad j=2.$

1.18. $\begin{vmatrix} 3 & 5 & 3 & 2 \\ 2 & 4 & 1 & 0 \\ 1 & -2 & 2 & 1 \\ 5 & 1 & -2 & 4 \end{vmatrix}$, $i=2,\quad j=4$ **1.19.** $\begin{vmatrix} 3 & 2 & 0 & -5 \\ 4 & 3 & -5 & 0 \\ 1 & 0 & -2 & 3 \\ 0 & 1 & -3 & 4 \end{vmatrix}$, $i=1,\quad j=2$

1.20. $\begin{vmatrix} 6 & 2 & 10 & 4 \\ 5 & 7 & -4 & 1 \\ 2 & 4 & -2 & -6 \\ 3 & 0 & -5 & 4 \end{vmatrix}$, $i=2, j=3.$ **1.21.** $\begin{vmatrix} -1 & 2 & 4 & 1 \\ 2 & 3 & 0 & 6 \\ 2 & 2 & 1 & 4 \\ 3 & 1 & 2 & -1 \end{vmatrix}$, $i=4, j=3.$

1.22. $\begin{vmatrix} 1 & 1 & -2 & 0 \\ 3 & 6 & -2 & 5 \\ 1 & 0 & 6 & 4 \\ 2 & 3 & 5 & -1 \end{vmatrix}$, $i=4,\ j=1.$ **1.23.** $\begin{vmatrix} 2 & 0 & -1 & -3 \\ 6 & 3 & -9 & 0 \\ 0 & 2 & -1 & 3 \\ 4 & 2 & 0 & 6 \end{vmatrix}$, $i=3,\ j=3.$

1.24. $\begin{vmatrix} -1 & 2 & 0 & 4 \\ 2 & -3 & 1 & 1 \\ 3 & -1 & 2 & 4 \\ 2 & 0 & 1 & 3 \end{vmatrix}$, $i=4,\ j=4.$ **1.25.** $\begin{vmatrix} 4 & 1 & 2 & 0 \\ -1 & 2 & 1 & -1 \\ 3 & 1 & 2 & 1 \\ 5 & 0 & 4 & 4 \end{vmatrix}$, $i=3,\ j=2.$

1.25. $\begin{vmatrix} 4 & 3 & -2 & -1 \\ 2 & 1 & -4 & 3 \\ 0 & 4 & 1 & -2 \\ 5 & 0 & 1 & -1 \end{vmatrix}$, $i=2,\ j=3.$

2-masala. A, B matritsalar va α, β sonlari berilgan. $\alpha A+\beta B$, AB, A^{-1} matritsalarni toping.

2.1. $A=\begin{pmatrix} 5 & 4 & 2 \\ 3 & 2 & 4 \\ 1 & 0 & 5 \end{pmatrix}$, $B=\begin{pmatrix} 5 & 4 & -5 \\ 3 & -7 & 1 \\ 1 & 2 & 2 \end{pmatrix}$, $\alpha=-1,\ \beta=4.$

2.2. $A=\begin{pmatrix} 3 & -1 & 0 \\ 3 & 5 & 1 \\ 4 & -7 & 5 \end{pmatrix}$, $B=\begin{pmatrix} -1 & 0 & 2 \\ 1 & -8 & 5 \\ 3 & 0 & 2 \end{pmatrix}$, $\alpha=-3, \beta=5.$

2.3. $A=\begin{pmatrix} 5 & -8 & -4 \\ 7 & 0 & -5 \\ 4 & 1 & 0 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 5 & 5 \\ 1 & 2 & 1 \\ 2 & -1 & -3 \end{pmatrix}$, $\alpha=5,\ \beta=-1.$

2.4. $A=\begin{pmatrix} 5 & -8 & -4 \\ 7 & 0 & -5 \\ 4 & 1 & 0 \end{pmatrix}$, $B=\begin{pmatrix} 1 & 5 & 5 \\ 1 & 2 & 1 \\ 2 & -1 & -3 \end{pmatrix}$, $\alpha=-3,\ \beta=1.$

2.5. $A=\begin{pmatrix}1 & 2 & 1\\ 1 & -2 & 4\\ 3 & -5 & 3\end{pmatrix}$, $B=\begin{pmatrix}7 & 5 & 1\\ 5 & 3 & -1\\ 1 & 2 & 3\end{pmatrix}$, $\alpha=-1,\ \beta=-3$.

2.6. $A=\begin{pmatrix}3 & 1 & 2\\ -1 & 0 & 2\\ 1 & 2 & 1\end{pmatrix}$, $B=\begin{pmatrix}0 & -1 & 2\\ 2 & 1 & 1\\ 3 & 7 & 1\end{pmatrix}$, $\alpha=1,\ \beta=1$.

2.7. $A=\begin{pmatrix}6 & 7 & 3\\ 3 & 1 & 0\\ 2 & 2 & 1\end{pmatrix}$, $B=\begin{pmatrix}2 & 0 & 5\\ 4 & -1 & 2\\ 4 & 3 & 7\end{pmatrix}$, $\alpha=1,\ \beta=3$.

2.8. $A=\begin{pmatrix}-2 & 3 & 4\\ 3 & -1 & -4\\ -1 & 2 & 2\end{pmatrix}$, $B=\begin{pmatrix}3 & 3 & 1\\ 0 & 6 & 2\\ 1 & 9 & 2\end{pmatrix}$, $\alpha=2,\ \beta=-2$.

2.9. $A=\begin{pmatrix}-3 & 4 & 2\\ 1 & 5 & 3\\ 0 & 1 & 2\end{pmatrix}$, $B=\begin{pmatrix}1 & 4 & 4\\ 1 & 3 & 2\\ 4 & 1 & 2\end{pmatrix}$, $\alpha=-5,\ \beta=1$.

2.10. $A=\begin{pmatrix}-1 & 0 & 2\\ 2 & 3 & 2\\ 3 & 7 & 1\end{pmatrix}$, $B=\begin{pmatrix}3 & 0 & 1\\ -3 & 1 & 7\\ 1 & 3 & 2\end{pmatrix}$, $\alpha=-1,\ \beta=4$.

2.11. $A=\begin{pmatrix}1 & 7 & 3\\ -4 & 9 & 4\\ 0 & 3 & 2\end{pmatrix}$, $B=\begin{pmatrix}6 & 5 & 2\\ 1 & 9 & 2\\ 4 & 5 & 2\end{pmatrix}$, $\alpha=-3,\ \beta=-2$.

2.12. $A=\begin{pmatrix}2 & 6 & 1\\ 1 & 3 & 2\\ 0 & 1 & 1\end{pmatrix}$, $B=\begin{pmatrix}4 & -3 & 2\\ -4 & 0 & 5\\ 3 & 2 & -3\end{pmatrix}$, $\alpha=1,\ \beta=2$.

2.13. $A=\begin{pmatrix}6 & 9 & 4\\ -1 & -1 & 1\\ 10 & 1 & 7\end{pmatrix}$, $B=\begin{pmatrix}1 & 1 & 1\\ 3 & 4 & 3\\ 0 & 5 & 2\end{pmatrix}$, $\alpha=5,\ \beta=2$.

2.14. $A=\begin{pmatrix}1 & 0 & 3\\ 3 & 1 & 7\\ 2 & 1 & 8\end{pmatrix}$, $B=\begin{pmatrix}3 & 5 & 4\\ -3 & 0 & 1\\ 5 & 6 & -4\end{pmatrix}$, $\alpha=-5,\ \beta=-2$.

2.15. $A=\begin{pmatrix}5 & 1 & -2\\ 1 & 3 & -1\\ 8 & 4 & -1\end{pmatrix}$, $B=\begin{pmatrix}3 & 5 & 5\\ 7 & 1 & 2\\ 1 & 6 & 0\end{pmatrix}$, $\alpha=-2,\ \beta=-2$.

2.16. $A=\begin{pmatrix}1 & -2 & 5\\ 3 & 0 & 6\\ 4 & 3 & 4\end{pmatrix}$, $B=\begin{pmatrix}-1 & -1 & 1\\ 2 & 3 & 3\\ 1 & -2 & -1\end{pmatrix}$, $\alpha=-1,\ \beta=-2$.

2. 17. $A=\begin{pmatrix}2 & -1 & -3\\ 8 & -7 & -6\\ -3 & 4 & 2\end{pmatrix}$, $B=\begin{pmatrix}2 & -1 & -2\\ 3 & -5 & 4\\ 1 & 2 & 1\end{pmatrix}$, $\alpha=1,\ \beta=2.$

2.18. $A=\begin{pmatrix}3 & 1 & 0\\ 4 & 3 & 2\\ 2 & 2 & -7\end{pmatrix}$, $B=\begin{pmatrix}3 & 1 & 0\\ 4 & 3 & 2\\ 2 & 2 & -7\end{pmatrix}$, $\alpha=2,\ \beta=5$.

2.19. $A=\begin{pmatrix}-3 & 4 & 0\\ 4 & 5 & 1\\ -2 & 3 & 3\end{pmatrix}$, $B=\begin{pmatrix}1 & 7 & -1\\ 0 & 2 & 6\\ 2 & -1 & 1\end{pmatrix}$, $\alpha=1,\ \beta=3$.

2.20. $A=\begin{pmatrix}-3 & 4 & -3\\ 1 & 2 & 3\\ 5 & 0 & -1\end{pmatrix}$, $B=\begin{pmatrix}2 & -2 & 0\\ 5 & 4 & 1\\ 1 & 1 & 2\end{pmatrix}$, $\alpha=4, \beta=5$.

2.21. $A=\begin{pmatrix}3 & 5 & -6\\ 2 & 4 & 3\\ -3 & 1 & 1\end{pmatrix}$, $B=\begin{pmatrix}2 & 8 & -5\\ -3 & -1 & 0\\ 4 & 5 & 3\end{pmatrix}$, $\alpha=3,\ \beta=2$.

2.22. $A=\begin{pmatrix}2 & -1 & 0\\ 3 & 3 & 1\\ 4 & -4 & -5\end{pmatrix}$, $B=\begin{pmatrix}-3 & 0 & -2\\ 1 & -6 & 3\\ 2 & 0 & 2\end{pmatrix}$, $\alpha=2,\quad \beta=-3.$

2.23. $A=\begin{pmatrix}2 & -1 & -4\\ 4 & -9 & 3\\ 2 & -7 & 1\end{pmatrix}$, $B=\begin{pmatrix}0 & 0 & -4\\ 5 & -6 & 4\\ 7 & -4 & 1\end{pmatrix}$, $\alpha=-5,\ \beta=1$.

2.24. $A=\begin{pmatrix}8 & 5 & -1\\ 1 & 5 & 3\\ 1 & 1 & 0\end{pmatrix}$, $B=\begin{pmatrix}4 & -7 & -6\\ 3 & 2 & -1\\ 0 & 1 & 2\end{pmatrix}$, $\alpha=-1, \beta=-2$.

2.25. $A=\begin{pmatrix} 2 & 1 & -1 \\ 2 & -1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$, $B=\begin{pmatrix} 3 & 6 & 0 \\ 2 & 4 & 6 \\ 1 & -2 & 3 \end{pmatrix}$, $\alpha=3$, $\beta=5$.

3-masala. Tenglamalar sistemalarini birgalikda bo‘lish-bo‘lmasligini Kroniker-Kapelli teoremasi bilan tekshiring. Birgalikda bo‘lgan sistemani Kramer formulalari orqali, matritsalar va Gauss usullari bilan yeching.

3.1. $\begin{cases} 3x_1 + x_2 + 2x_3 = 1, \\ x_1 + 3x_2 + 2x_3 = 7, \\ 2x_1 + x_2 + 3x_3 = 6. \end{cases}$

3.2. $\begin{cases} 2x_1 - x_2 + 2x_3 = 3, \\ x_1 + x_2 + 2x_3 = -4, \\ 4x_1 + x_2 + 4x_3 = -3. \end{cases}$

3.3. $\begin{cases} 3x_1 + x_2 - 2x_3 = 6, \\ 5x_1 - 3x_2 + 2x_3 = -4, \\ 4x_1 - 2x_2 - 3x_3 = -2. \end{cases}$

3.4. $\begin{cases} 3x_1 - x_2 + x_3 = -11, \\ 5x_1 + x_2 + 2x_3 = 8, \\ x_1 + 2x_2 + 4x_3 = 16. \end{cases}$

3.5. $\begin{cases} x_1 - 3x_2 - x_3 = 1, \\ 2x_1 + x_2 + x_3 = -7, \\ 2x_1 - x_2 - 3x_3 = 5. \end{cases}$

3.6. $\begin{cases} x_1 + 2x_2 + x_3 = 8, \\ 4x_1 - 3x_2 - 2x_3 = -1, \\ 2x_1 - x_2 + 3x_3 = 1. \end{cases}$

3.7. $\begin{cases} 2x_1 + 3x_2 - x_3 = 2, \\ x_1 - x_2 + 3x_3 = -4, \\ 3x_1 + 5x_2 + x_3 = 4. \end{cases}$

3.8. $\begin{cases} 4x_1 + 2x_2 - 3x_3 = -2, \\ x_1 + x_2 + 2x_3 = 5, \\ 3x_1 + 2x_2 - 2x_3 = -1. \end{cases}$

3.9. $\begin{cases} 2x_1 - x_2 + 5x_3 = 27, \\ 5x_1 + 2x_2 + 13x_3 = 70, \\ 3x_1 - x_3 = -2. \end{cases}$

3.10. $\begin{cases} 4x_1 + x_2 - 3x_3 = -6, \\ 8x_1 + 3x_2 - 6x_3 = -15, \\ x_1 + x_2 - x_3 = -4. \end{cases}$

3.11. $\begin{cases} x_1 + 3x_2 - x_3 = 0, \\ 4x_1 - 5x_2 + x_3 = 7, \\ 3x_1 - x_2 + 4x_3 = -4. \end{cases}$

3.12. $\begin{cases} 2x_1 - x_2 + 3x_3 = 1, \\ 3x_1 + 2x_2 - 2x_3 = 1, \\ x_1 - 3x_2 + 4x_3 = 3. \end{cases}$

3.13. $\begin{cases} 4x_1 - 7x_2 = 1, \\ 2x_1 + x_2 - 3x_3 = -1, \\ 3x_1 + 5x_3 = 16. \end{cases}$

3.14. $\begin{cases} 5x_1 + 7x_2 - x_3 = 1, \\ x_1 + 7x_3 = 6, \\ 2x_1 - 4x_2 + 5x_3 = -1. \end{cases}$

3.15. $\begin{cases} 3x_1 + 2x_2 - x_3 = 6, \\ x_1 + 3x_2 + 2x_3 = 9, \\ 4x_1 - 5x_2 + x_3 = 5. \end{cases}$

3.16. $\begin{cases} 2x_1 + x_2 - 3x_3 = 11, \\ 4x_1 + 8x_3 = -4, \\ 5x_1 - 6x_2 = 21. \end{cases}$

3.17. $\begin{cases} 3x_1 - x_2 + 3x_3 = 2, \\ 3x_1 + 6x_2 = 3, \\ 2x_1 - 5x_3 = -12. \end{cases}$

3.18. $\begin{cases} 2x_1 + 4x_2 - x_3 = 7, \\ 4x_1 - x_2 + 5x_3 = -11, \\ x_1 + 3x_2 - x_3 = 6. \end{cases}$

3.19. $\begin{cases} 3x_1 + 5x_2 - x_3 = 1, \\ 2x_1 + x_2 + x_3 = -3, \\ x_1 + 4x_2 - 3x_3 = 2. \end{cases}$

3.20. $\begin{cases} 5x_1 + x_2 - 2x_3 = 7, \\ 2x_1 - x_2 + 3x_3 = 2, \\ 2x_1 + 7x_3 = 16. \end{cases}$

3.21. $\begin{cases} 2x_1 - x_2 - 3x_3 = -9, \\ x_1 + 5x_2 + x_3 = 20, \\ 3x_1 + 4x_2 + 2x_3 = 15. \end{cases}$

3.22. $\begin{cases} 4x_1 - x_2 - x_3 = 10, \\ 2x_1 + 6x_2 = 38, \\ 3x_1 - 7x_3 = 5. \end{cases}$

3.23. $\begin{cases} 3x_1 - x_2 + x_3 = 12, \\ 5x_1 + x_2 + 2x_3 = 3, \\ x_1 + 2x_2 + 4x_3 = 6. \end{cases}$

3.24. $\begin{cases} 2x_1 - 3x_2 + 4x_3 = 3, \\ 3x_1 + x_2 - 5x_3 = 10, \\ 4x_1 + x_2 + 6x_3 = 1. \end{cases}$

3.25. $\begin{cases} 7x_1 - 5x_2 + x_3 = -33, \\ 4x_1 + x_3 = -7, \\ 2x_1 + 3x_2 + 4x_3 = 12. \end{cases}$

4-masala. Bir jinsli tenglamalar sistemasini yeching.

4.1. $\begin{cases} 2x_1 - 3x_2 + x_3 = 0, \\ 5x_2 + 2x_3 = 0, \\ 4x_1 - x_2 + 4x_3 = 0. \end{cases}$

4.2. $\begin{cases} 4x_1 - 2x_2 + x_3 = 0, \\ 3x_1 + x_2 - 3x_3 = 0, \\ 2x_1 + 4x_2 - 7x_3 = 0. \end{cases}$

4.3. $\begin{cases} 2x_1 + 5x_2 - x_3 = 0, \\ 2x_1 + 11x_2 - 5x_3 = 0, \\ 2x_1 - x_2 + 3x_3 = 0. \end{cases}$

4. 4. $\begin{cases} 5x_1 - x_2 - 3x_3 = 0, \\ 3x_1 + 2x_2 + x_3 = 0, \\ x_1 + 5x_2 + 5x_3 = 0. \end{cases}$

4.5. $\begin{cases} 4x_1 + x_2 - 3x_3 = 0, \\ 5x_1 + 2x_2 - x_3 = 0, \\ x_1 + x_2 + 2x_3 = 0. \end{cases}$

4. 6. $\begin{cases} 5x_1 + x_2 - 4x_3 = 0, \\ 2x_1 - 3x_2 + 2x_3 = 0, \\ x_1 - 10x_2 + 10x_3 = 0. \end{cases}$

4. 7. $\begin{cases} 2x_1 - x_2 - 3x_3 = 0, \\ 3x_1 + 2x_2 - 3x_3 = 0, \\ x_1 - 4x_2 - 3x_3 = 0. \end{cases}$

4. 8. $\begin{cases} 4x_1 - x_2 + 2x_3 = 0, \\ 2x_1 - 3x_2 - x_3 = 0, \\ -2x_1 + 8x_2 + 5x_3 = 0. \end{cases}$

4. 9. $\begin{cases} 4x_1 + x_2 - 3x_3 = 0, \\ x_1 - 2x_2 + x_3 = 0, \\ 5x_1 - x_2 - 2x_3 = 0. \end{cases}$

4. 10. $\begin{cases} 2x_1 + 3x_2 - x_3 = 0, \\ 5x_1 - x_2 + 2x_3 = 0, \\ x_1 - 7x_2 + 4x_3 = 0. \end{cases}$

4. 11. $\begin{cases} 3x_1 - 2x_2 + x_3 = 0, \\ 4x_1 - x_2 - 2x_3 = 0, \\ 2x_1 - 3x_2 + 4x_3 = 0. \end{cases}$

4. 12. $\begin{cases} 3x_1 + x_2 - 5x_3 = 0, \\ 2x_1 + x_2 + 3x_3 = 0, \\ 4x_1 + x_2 - 13x_3 = 0. \end{cases}$

4. 13. $\begin{cases} 2x_1 + 6x_2 - 3x_3 = 0, \\ 3x_1 - 2x_2 + x_3 = 0, \\ x_1 + 14x_2 - 7x_3 = 0. \end{cases}$

4. 14. $\begin{cases} 3x_1 - 2x_2 + x_3 = 0, \\ 7x_1 - 9x_2 + 5x_3 = 0, \\ 2x_1 + 3x_2 - 2x_3 = 0. \end{cases}$

4. 15. $\begin{cases} x_1 - 2x_2 - 3x_3 = 0, \\ x_1 + 3x_2 - 5x_3 = 0, \\ 2x_1 + x_2 - 8x_3 = 0. \end{cases}$

4 .16. $\begin{cases} 3x_1 + x_2 - 2x_3 = 0, \\ x_1 + 3x_2 - 5x_3 = 0, \\ 5x_1 - x_2 + x_3 = 0. \end{cases}$

4. 17. $\begin{cases} 2x_1 + 4x_2 - 5x_3 = 0, \\ 3x_1 - 3x_2 + 4x_3 = 0, \\ x_1 + 11x_2 - 14x_3 = 0. \end{cases}$

4. 18. $\begin{cases} 5x_1 - 4x_2 + x_3 = 0, \\ 3x_1 + 2x_2 - x_3 = 0, \\ x_1 + 8x_2 - 3x_3 = 0. \end{cases}$

4. 19. $\begin{cases} 5x_1 - x_2 - 2x_3 = 0, \\ 3x_1 - 4x_2 + x_3 = 0, \\ 2x_1 + 3x_2 - 3x_3 = 0. \end{cases}$

4. 20. $\begin{cases} 5x_1 - 5x_2 - 4x_3 = 0, \\ 4x_1 - 4x_2 - 9x_3 = 0, \\ 3x_1 - 3x_2 - 14x_3 = 0. \end{cases}$

4. 21. $\begin{cases} 3x_1 - x_2 + 2x_3 = 0, \\ 4x_1 + 3x_3 = 0, \\ x_1 + x_2 + x_3 = 0. \end{cases}$

4. 22. $\begin{cases} 3x_1 + x_2 - 4x_3 = 0, \\ x_1 + 2x_2 - x_3 = 0, \\ x_1 + 7x_2 = 0. \end{cases}$

4.23. $\begin{cases} 3x_1 + 5x_2 - x_3 = 0, \\ 2x_1 + 11x_2 - 5x_3 = 0, \\ 4x_1 - x_2 + 3x_3 = 0. \end{cases}$

4. 24. $\begin{cases} 2x_1 + 3x_3 = 0, \\ x_1 - x_2 + 2x_3 = 0, \\ x_1 + x_2 + x_3 = 0. \end{cases}$

4. 25. $\begin{cases} 4x_1 + x_2 - 3x_3 = 0, \\ 2x_1 - 3x_2 + x_3 = 0, \\ 2x_1 - 10x_2 + 6x_3 = 0. \end{cases}$

5-masala. ABC uchburchak uchlarining koordinatalari berilgan: a) C uchdan tushirilgan balandlik tenglamasini tuzing va uning uzunligini toping; b) B uchdan o‘tkazilgan mediana tenglamasini tuzing va uchburchak medianalarining kesishish nuqtalarini toping; c) A burchakning radian qiymatini hisoblang va uning bissektrisasi tenglamasini tuzing.

5.1. $A(1;2)$, $B(9;8)$, $C(6;14)$.

5.2. $A(2;-3)$, $B(-3;9)$, $C(6;0)$.

5.3 $A(-1;-2)$, $B(7;4)$, $C(4;10)$.

5.4. $A(1;-1)$, $B(9;5)$, $C(6;11)$.

5.5. $A(1;-4)$, $B(-4;8)$, $C(5;-1)$.

5.6. $A(-1;1)$, $B(7;7)$, $C(4;13)$.

5.7. $A(5;-2)$, $B(8;2)$, $C(-7;3)$.

5.8. $A(2;-4)$, $B(14;1)$, $C(-2;-1)$.

5.9. $A(6;0)$, $B(9;4)$, $C(-6;5)$.

5.10. $A(8;2)$, $B(-4;7)$, $C(14;10)$.

5.11. $A(-1;-6)$, $B(-6;6)$, $C(3;-3)$.

5.12. $A(4;-1)$, $B(7;-5)$, $C(-8;4)$.

5.13. $A(12;0)$, $B(0;5)$, $C(18;8)$.

5.14. $A(1;-2)$, $B(-11;3)$, $C(7;6)$.

5.15. $A(3;4)$, $B(15;9)$, $C(-1;7)$.

5.16. $A(-1;2)$, $B(7;8)$, $C(4;14)$.

5.17. $A(1;1)$, $B(9;7)$, $C(6;13)$.

5.18. $A(14;-6)$, $B(26;-1)$, $C(20;2)$.

5.19. $A(2;-1)$, $B(10;5)$, $C(7;11)$.

5.20. $A(5;-3)$, $B(17;2)$, $C(1;0)$.

5.21. $A(-2;1)$, $B(6;7)$, $C(3;13)$.

5.22. $A(2;-1)$, $B(-10;4)$, $C(8;7)$.

5.23. $A(-1;-1)$, $B(7;5)$, $C(4;11)$.

5.24. $A(-2;-6)$, $B(10;-1)$, $C(-6;-3)$.

5.25. $A(3;-7)$, $B(-2;5)$, $C(7;-4)$.

6-masala. 1. (1-11) Har bir $M(x;y)$ nuqtasidan berilgan $A(x_1;y_1)$ va $B(x_2;y_2)$ nuqtalargacha bo‘lgan masofalar nisbati a ga teng bo‘lgan chiziq tenglamasini tuzing.

2. (11-25) Har bir $M(x;y)$ nuqtasidan berilgan $A(x_1;y_1)$ nuqtagacha va $x=b$ to‘g‘ri chiziqqacha bo‘lgan masofalar nisbati m ga teng bo‘lgan chiziq tenglamasini tuzing.

6.1. $A(4;1)$, $B(-2;-1)$, $a=4$.

6.2. $A(5;7)$, $B(-2;1)$, $a=4$.

6.3. $A(-3;3)$, $B(5;1)$, $a=\frac{1}{3}$.

6.4. $A(2;-4)$, $B(3;5)$, $a=\frac{2}{3}$.

6.5. $A(1;6)$, $B(4;-2)$, $a=2$.

6.6. $A(3;-2)$, $B(4;6)$, $a=\frac{3}{5}$.

6.7. $A(6;0)$, $B(0;-3)$, $a=2$.

6.8. $A(-4;0)$, $B(0;0)$, $a=3$.

6.9. $A(4;-2)$, $B(1;6)$, $a=2$.

6.10. $A(2;1)$, $B(-2;2)$, $a=4$.

6.11. $A(-3;3)$, $B(5;1)$, $a=3$.

6.12. $A(6;1)$, $x=-5$, $m=\frac{1}{3}$.

6.13. $A(-1;2)$, $x=9$, $m=\frac{1}{4}$.

6.14. $A(1;0)$, $x=8$, $m=\frac{1}{5}$.

6.15. $A(0;5)$, $x=3$, $m=\frac{1}{2}$.

6.16. $A(2;1)$, $x=-5$, $m=3$.

6.17. $A(-3;4)$, $x=3$, $m=3$.

6.18. $A(2;0)$, $x=-\frac{5}{2}$, $m=\frac{4}{5}$.

6.19. $A(2;0)$, $x=-\frac{8}{5}$, $m=\frac{5}{4}$.

6.20. $A(-1;0)$, $x=-4$, $m=\frac{1}{2}$.

6.21. $A(4;0)$, $x=-2$, $m=\frac{1}{2}$.

6.22. $A(3;0)$, $x=\frac{9}{2}$, $m=\frac{2}{3}$.

6.23. $A(1;3)$, $x=-6$, $m=\frac{1}{2}$.

6.24. $A(1;5)$, $x=-1$, $m=\frac{1}{4}$.

6.25. $A(3;0)$, $B(-6;0)$, $a=\frac{1}{2}$.

*7-**masala.*** $ABCD$ piramidaning uchlari berilgan: a) AB qirra tenglamasini tuzing; b) ABC yoq tenglamasini tuzing; c) D uchdan ABC yoqqa tushirilgan balandlik tenglamasini tuzing va uning uzunligini toping; d) C uchdan o‘tuvchi AB qirraga parallel to‘g‘ri chiziq tenglamasini tuzing; e) D uchdan o‘tuvchi AB qirraga perpendikular tekislik tenglamasini tuzing; f) AD qirra bilan ABC yoq orasidagi burchak sinusini toping; g) ABC va ABD yoqlar orasidagi burchak kosinusini toping.

7.1. $A(3;5;3)$, $B(8;7;4)$, $C(5;10;4)$, $D(4;7;8)$.

7.2. $A(6;6;5)$, $B(4;9;5)$, $C(4;6;11)$, $D(6;9;3)$.

7.3. $A(3;2;2)$, $B(5;-3;2)$, $C(5;-3;-1)$, $D(2;-3;7)$.

7.4. $A(0;4;5)$, $B(3;-2;1)$, $C(-4;5;6)$, $D(3;3;-2)$.

7.5. $A(1;-1;3)$, $B(6;5;8)$, $C(3;5;8)$, $D(8;4;1)$.

7.6. $A(1;-2;7)$, $B(4;2;10)$, $C(2;-3;5)$, $D(5;3;7)$.

7.7. $A(4;2;7)$, $B(1;2;0)$, $C(3;5;7)$, $D(2;-3;5)$.

7.8. $A(2;3;5)$, $B(5;3;-7)$, $C(1;2;7)$, $D(5;2;0)$.

7.9. $A(5;3;7)$, $B(-2;3;5)$, $C(4;2;7)$, $D(1;-2;7)$.

7.10. $A(3;1;4)$, $B(-1;6;1)$, $C(-1;1;6)$, $D(0;4;-1)$.

7.11. $A(3;-1;2)$, $B(-1;0;1)$, $C(1;7;3)$, $D(9;5;8)$.

7.12. $A(3;5;4)$, $B(5;8;4)$, $C(1;2;-2)$, $D(-1;3;2)$.

7.13. $A(2;4;3)$, $B(1;1;5)$, $C(4;9;3)$, $D(-3;6;7)$.

7.14. $A(9;5;5)$, $B(-3;7;1)$, $C(5;7;8)$, $D(6;0;2)$.

7.15. $A(2;9;6)$, $B(2;8;2)$, $C(9;8;6)$, $D(7;9;3)$.

7.16. $A(2;5;-1)$, $\dfrac{x+3}{2}=\dfrac{y-5}{4}=\dfrac{z}{-1}$.

7.17. $A(1;8;6)$, $B(5;2;2)$, $C(5;7;6)$, $D(4;8;-1)$.

7.18. $A(0;7;1)$, $B(2;-1;5)$, $C(1;6;3)$, $D(3;-9;-8)$.

7.19. $A(5;5;4)$, $B(1;-1;4)$, $C(3;5;1)$, $D(5;8;-3)$.

7.20. $A(6;1;1)$, $B(1;6;6)$, $C(4;2;0)$, $D(1;2;6)$.

7.21. $A(7;5;3)$, $B(9;4;4)$, $C(4;5;7)$, $D(7;9;6)$.

7.22. $A(6;8;2)$, $B(5;4;7)$, $C(2;8;2)$, $D(7;3;7)$.

7.23. $A(4;2;5)$, $B(0;6;1)$, $C(0;2;7)$, $D(1;4;0)$.

7.24. $A(4;4;9)$, $B(7;10;3)$, $C(2;8;4)$, $D(9;6;9)$.

7.25. $A(4;6;5)$, $B(6;9;4)$, $C(2;3;5)$, $D(7;5;9)$.

8-masala. Sirt turini aniqlang va shaklini chizing.

8.1. a) $5x^2+y^2-3z^2=0$; b) $z^2=2y^2+4$.

8.2. a) $x^2+4z^2+6y=0$; b) $4x^2+3z^2=12$.

8.3. a) $8x^2-y^2+4z^2+32=0$; b) $3y^2+2z^2=6$.

8.4. a) $6x^2+5y^2-10z^2-30=0$; b) $5x^2-4z^2=6$.

8.5. a) $2x^2+6y^2=3z$; b) $3x^2+6z^2=18$.

8.6. a) $2x^2-3y^2-5z^2+30=0$; b) $3z^2-2x=6$.

8.7. a) $x^2-6y^2+z^2-124=0$; b) $2x^2-3z^2=6$.

8.8. a) $3z^2+9y^2-x=0$; b) $3x^2+5z^2=15$.

8.9. a) $y-4z^2=3x^2$; b) $x^2-4z^2=4$.

8.10. a) $3x^2+5y^2-4z=0$; b) $5x^2+4z^2=20$.

8.11. a) $9x^2+12y^2+4z^2-72=0$; b) $4x^2-3y^2=12$.

8.12. a) $10x^2-9y^2-15z^2-9=0$; b) $y^2=2z^2+z$.

8.13. a) $6z^2-3y^2-2x^2-18=0$; b) $4y^2-5z^2=20$.

8.14. a) $3x^2-9y^2+z^2+27=0;$ b) $x^2-4z^2=10.$

8.15. a) $4x^2+z^2-2y=0;$ b) $y^2=x+3$.

8.16. a) $2y^2+6z=3x^2;$ b) $z^2=x-4.$

8.17. a) $4x^2-12y^2+3z^2-24=0;$ b) $3x^2+z^2=30$.

8.18. a) $2x^2+4y^2-5z^2=0;$ b) $7x^2-5z^2=35.$

8.19. a) $7x^2+2y^2+6z^2-42=0;$ b) $x^2+4z^2=4$.

8.20. a) $4x^2+9y^2-36z^2=0$; b) $2y^2-3x=12$.

8.21. a) $4x^2+4y^2+5z^2-20=0$; b) $9x^2+4y^2=36$.

8.22. a) $5x^2+5y^2-6z^2-30=0$; b) $z^2=4y^2-3$.

8.23. a) $4x^2-3y^2+2z^2-24=0$; b) $x^2-y^2=2y$.

8.24. a) $8x^2-y^2-2z^2-32=0;$ b) $2x^2+3z^2=6-12z$.

8.25. a) $2x^2-2y^2-5z^2-10=0$; b) $x^2+2x=z^2+1$.

9-masala. Sonli ketma-ketliklarning limitini toping.

9.1. a) $x_n=\sqrt{n^2-5n+6}-n;$ b) $x_n=\dfrac{(n+2)!+(n+3)!}{(n+4)!}.$

9.3. a) $x_n=\sqrt{n^2-2n+6}-\sqrt{n^2+2n-6};$ b) $x_n=\dfrac{1+3+5+\cdots+(2n-1)}{\sqrt{2n^2+n-2}}.$

9.4. a) $x_n=\sqrt[3]{5+8n^3}-2n$; b) $x_n=\dfrac{1}{n^2}(1+2+3+\cdots+n).$

9.5. a) $x_n=\sqrt{n^4+3}-\sqrt{n^4-2};$ b) $x_n=\dfrac{2+4+6+\cdots+2n}{n+5}-n.$

9.6. a) $x_n=n-\sqrt{n(n-1)};$ b) $x_n=\dfrac{5}{6}+\dfrac{13}{36}+\cdots+\dfrac{2^n+3^n}{6^n}.$

9.7. a) $x_n = n\cdot(\sqrt[3]{5+8n^3} - 2n)$; b) $x_n = \dfrac{1+2+3+\dots+n}{\sqrt[3]{n^6+n}}$.

9.8. a) $x_n = n - \sqrt[3]{n^3-3}$; b) $x_n = \dfrac{2-5+4-7+\dots+2n-(2n+3)}{n+5}$.

9.9. a) $x_n = \sqrt{n}\cdot(\sqrt{n+3}-\sqrt{n-2})$; b) $x_n = \dfrac{n!}{(n+1)!-n!}$.

9.10. a) $x_n = \sqrt{n+2}\cdot(\sqrt{n+4}-\sqrt{n-3})$; b) $x_n = \dfrac{1}{1\cdot 2}+\dfrac{1}{2\cdot 3}+\dots+\dfrac{1}{n(n+1)}$.

9.11. a) $x_n = \sqrt[3]{n^2-n^3}+n$; b) $x_n = \dfrac{2^n+3^n}{2^{n+1}+3^{n+1}}$.

9.12. a) $x_n = n-\sqrt{(n-2)(n+3)}$; b) $x_n = \dfrac{1}{1\cdot 4}+\dfrac{1}{4\cdot 7}+\dots+\dfrac{1}{(3n-2)(3n+1)}$.

9.13. a) $x_n = \sqrt{n(n+2)}-\sqrt{n^2-2n+3}$; b) $x_n = \dfrac{1}{1\cdot 7}+\dfrac{1}{3\cdot 9}+\dots+\dfrac{1}{(2n-1)(2n+5)}$.

9.14. a) $x_n = n\sqrt{n}-\sqrt{n(n+2)(n+3)}$; b) $x_n = \dfrac{1+2+3+\dots+n}{\sqrt{8n^2-1}}$.

9.15. a) $x_n = \sqrt{n^5-8}-n\sqrt{n(n^2+5)}$; b) $x_n = \dfrac{1+\frac{1}{3}+\frac{1}{3^2}+\dots+\frac{1}{3^n}}{1+\frac{1}{2}+\frac{1}{2^2}+\dots+\frac{1}{2^n}}$.

9.16. a) $x_n = n^2\cdot(\sqrt[3]{5+n^3}-\sqrt[3]{3+n^3})$; b) $x_n = \dfrac{1-2+3-4+\dots+(2n-1)-2n}{\sqrt{2+n^2}}$.

9.17. a) $x_n = \sqrt[3]{(n+2)^2}-\sqrt[3]{(n-2)^2}$; b) $x_n = \dfrac{3-n^2+2\sqrt{n}}{2+7+12+\dots+(5n-3)}$.

9.18. a) $x_n = n^2-\sqrt{n^4+n^2+1}$; b) $x_n = \dfrac{\sqrt[3]{3-n^3}+n^2}{1+3+5+\dots+(2n-1)}$.

9.19. a) $x_n = \sqrt{n^2-n+2}-\sqrt{n^2+n-1}$; b) $x_n = \dfrac{1}{1\cdot 3}+\dfrac{1}{3\cdot 5}+\dots+\dfrac{1}{(2n-1)(2n+1)}$.

9.20. a) $x_n = \sqrt{n^4-2}-\sqrt{n^4+3}$; b) $x_n = \dfrac{(3n-1)!+(3n+1)!}{3n!(n+1)}$.

9.21. a) $x_n = \sqrt{n^2+4}-\sqrt{n+n^2}$; b) $x_n = \dfrac{3n+1}{3}-\dfrac{2+5+8+\dots+(3n-1)}{2n+3}$.

9.22. a) $x_n=\sqrt{n^4+3n^2+1}-n^2$; b) $x_n=\dfrac{1+4+7+\cdots+(3n-2)}{\sqrt{n^4-n^2-1}}$.

9.23. a) $x_n=\sqrt[3]{n}\cdot(\sqrt[3]{n^2}-\sqrt[3]{n(n-1)})$; b) $x_n=\dfrac{3+5+7+\cdots+(2n+3)}{n\sqrt{n^2-1}}$.

9.24. a) $x_n=\sqrt{n^3+8}\cdot(\sqrt[3]{n^3+2}-\sqrt[3]{n^2-1})$; b) $x_n=\dfrac{5^n-2^n}{5^{n-1}+2^n}$.

9.25. a) $x_n=2n-\sqrt[3]{3+8n^3}$; b) $x_n=\dfrac{7}{10}+\dfrac{29}{100}+\dfrac{133}{1000}+\cdots+\dfrac{5^n+2^n}{10^n}$.

10-masala. Limitlarni toping:

10.1. $\lim\limits_{x\to5}\dfrac{3x^2-6x-45}{2x^2-3x-35}$. $\lim\limits_{x\to0}\dfrac{\sqrt{x^2+2}-\sqrt{2}}{\sqrt{x^2+1}-1}$.

$\lim\limits_{x\to0}\dfrac{\cos x-\cos^2 x}{5x^2}$. $\lim\limits_{x\to\infty}\left(\dfrac{2x-1}{2x+3}\right)^{2x-1}$.

10.2. $\lim\limits_{x\to-1}\dfrac{x^3-3x-2}{x^2-4x-5}$. $\lim\limits_{x\to1}\dfrac{3x^2-4x+1}{\sqrt{3+2x}-\sqrt{x+4}}$.

$\lim\limits_{x\to0}\dfrac{1-\cos5x}{4x^2}$. $\lim\limits_{x\to\infty}\left(\dfrac{x+5}{x+9}\right)^{-4x}$.

10.3. $\lim\limits_{x\to2}\dfrac{x^3-2x-4}{x^2-11x+18}$. $\lim\limits_{x\to2}\dfrac{\sqrt{4x+1}-3}{x^3-8}$.

$\lim\limits_{x\to0}\dfrac{tg4x}{3\sin5x}$. $\lim\limits_{x\to\infty}\left(\dfrac{2-3x}{5-3x}\right)^{2x}$.

10.4. $\lim\limits_{x\to1}\dfrac{3x^4-x^2-2}{2x^4-x-1}$. $\lim\limits_{x\to3}\dfrac{\sqrt{3x}-x}{x^3-27}$.

$\lim\limits_{x\to0}\dfrac{tg2x-\sin2x}{3x^2}$. $\lim\limits_{x\to\infty}\left(\dfrac{x+5}{x-7}\right)^{2x+3}$.

10.5. $\lim\limits_{x\to-2}\dfrac{4x^2+7x-2}{3x^2+8x+4}$. $\lim\limits_{x\to4}\dfrac{2-\sqrt{x}}{\sqrt{6x+1}-5}$.

$\lim\limits_{x\to0}\dfrac{1-\cos4x}{x\cdot\sin x}$. $\lim\limits_{x\to\infty}\left(\dfrac{5x-2}{5x+1}\right)^{-x}$.

10.6. $\lim\limits_{x\to4}\dfrac{3x^2-13x+4}{x^2-x-12}$. $\lim\limits_{x\to5}\dfrac{\sqrt{x+4}-3}{\sqrt{x-1}-2}$.

$$\lim_{x\to 0}\frac{\cos^3 x-\cos x}{1-\cos 3x}.$$ $$\lim_{x\to\infty}\left(\frac{x^2+3}{x^2}\right)^{2x+3}.$$

10.7. $\lim\limits_{x\to 1}\dfrac{x^4+4x^2-5}{x^3+2x^2-x-2}.$ $\lim\limits_{x\to 7}\dfrac{\sqrt{x-3}-2}{\sqrt{x+2}-3}.$

$$\lim_{x\to 0}\frac{1-\cos 8x}{1-\cos 4x}.$$ $$\lim_{x\to\infty}\left(\frac{3x-1}{3x+4}\right)^{4x-1}.$$

10.8. $\lim\limits_{x\to 1}\dfrac{8x^4-6x^2-x-1}{x^3-3x^2+2}.$ $\lim\limits_{x\to 8}\dfrac{\sqrt{9+2x}-5}{2-\sqrt[3]{x}}.$

$$\lim_{x\to 0}\frac{1-\cos^2 x}{x\cdot tgx}.$$ $$\lim_{x\to\infty}\left(\frac{5-2x}{3-2x}\right)^{-x+3}.$$

10.9. $\lim\limits_{x\to 2}\dfrac{x^3-3x^2+4}{3x^2-x-10}.$ $\lim\limits_{x\to 9}\dfrac{\sqrt{2x+7}-\sqrt{3x-2}}{x^2-10x+9}.$

$$\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{tgx}\right).$$ $$\lim_{x\to\infty}\left(\frac{4x-1}{4x+1}\right)^{3x}.$$

10.10. $\lim\limits_{x\to -1}\dfrac{3x^3-2x+1}{4x^3+2x^2-x+1}.$ $\lim\limits_{x\to -1}\dfrac{\sqrt{x+3}-\sqrt{5+3x}}{4x^2+3x-1}.$

$$\lim_{x\to 0}\frac{\sqrt{1+\sin^2 x}-1}{1-\cos 2x}.$$ $$\lim_{x\to\infty}\left(\frac{1+2x}{3+2x}\right)^{-x}.$$

10.11. $\lim\limits_{x\to -3}\dfrac{2x^2+11x+15}{3x^2+5x-12}.$ $\lim\limits_{x\to -2}\dfrac{x^2-x-6}{\sqrt{2-x}-\sqrt{x+6}}.$

$$\lim_{x\to 1}(1-x)tg\frac{\pi x}{2}.$$ $$\lim_{x\to\infty}\left(\frac{5x+8}{x-2}\right)^{x+4}.$$

10.12. $\lim\limits_{x\to 4}\dfrac{3x^2-2x-40}{x^2-3x-4}.$ $\lim\limits_{x\to -3}\dfrac{2x^2-x-21}{\sqrt{x+10}-\sqrt{4-x}}.$

$$\lim_{x\to\frac{\pi}{2}}\frac{1-\sin x}{\pi-2x}.$$ $$\lim_{x\to-\infty}\left(\frac{2x-1}{4x+1}\right)^{2x-1}.$$

10.13. $\lim\limits_{x\to 7}\dfrac{x^2-5x-14}{2x^2-19x+35}.$ $\lim\limits_{x\to -4}\dfrac{4-\sqrt{x+20}}{x^3+64}.$

$$\lim_{x\to\frac{\pi}{4}}\frac{1-\sin 2x}{4x-\pi}.$$ $$\lim_{x\to -1}(4x+5)^{\frac{3x}{x^2-1}}.$$

10.14. $\lim\limits_{x\to -1}\dfrac{x^2-x-2}{x^3+1}$. $\lim\limits_{x\to -5}\dfrac{\sqrt{3x+17}-\sqrt{7+x}}{x^2+4x-5}$.

$\lim\limits_{x\to 0}\dfrac{arctg2x}{tg3x}$. $\lim\limits_{x\to 1}(4-3x)^{\frac{x}{x^2-1}}$.

10.15. $\lim\limits_{x\to 3}\dfrac{3x^2-7x-6}{2x^2-7x+3}$. $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt[3]{1+x}-\sqrt[3]{1-x}}$.

$\lim\limits_{x\to 0}\dfrac{\sin x+\sin 3x}{\arcsin x}$. $\lim\limits_{x\to \infty}(2x+3)[\ln(x+2)-\ln x]$.

10.16. $\lim\limits_{x\to 2}\dfrac{x^3-8}{2x^2-9x+10}$. $\lim\limits_{x\to 1}\dfrac{\sqrt{x+8}-\sqrt{4x+5}}{3x^2+4x-7}$.

$\lim\limits_{x\to 0}\dfrac{\sin^2 2x-\sin^2 x}{3x^2}$. $\lim\limits_{x\to 2}(2x-3)^{\frac{3x}{x-2}}$.

10.17. $\lim\limits_{x\to 1}\dfrac{x^3+x-2}{x^2-x^2+x-1}$. $\lim\limits_{x\to 2}\dfrac{3x^2-2x-8}{\sqrt{2x+1}-\sqrt{9-2x}}$.

$\lim\limits_{x\to 0}\dfrac{1-\cos^2 2x}{x\cdot arctgx}$. $\lim\limits_{x\to -\infty}(2x-1)[\ln(1-3x)-\ln(2-3x)]$.

10.18. $\lim\limits_{x\to 2}\dfrac{x^2-4}{3x^2+x-10}$. $\lim\limits_{x\to 4}\dfrac{\sqrt{2x+1}-3}{\sqrt{x-2}-\sqrt{2}}$.

$\lim\limits_{x\to \frac{\pi}{4}}\dfrac{\cos x-\sin x}{1-tgx}$. $\lim\limits_{x\to \infty}\left(\dfrac{1-x}{2-x}\right)^{3x-1}$.

10.19. $\lim\limits_{x\to 6}\dfrac{2x^2-11x-6}{3x^2-20x+12}$. $\lim\limits_{x\to 5}\dfrac{\sqrt{2x+1}-\sqrt{x+6}}{x^2-8x+15}$.

$\lim\limits_{x\to 0}\dfrac{1-\cos^2 x}{x\cdot \arcsin x}$. $\lim\limits_{x\to \infty}\left(\dfrac{x+1}{3x-1}\right)^{2x+1}$.

10.20. $\lim\limits_{x\to 3}\dfrac{6+x-x^2}{x^3-27}$. $\lim\limits_{x\to 8}\dfrac{\sqrt{5x+9}-7}{2-\sqrt[3]{x}}$.

$\lim\limits_{x\to \frac{\pi}{2}}\dfrac{1-\sin x}{(\pi/2-x)^2}$. $\lim\limits_{x\to -\infty}\left(\dfrac{4+3x}{5+x}\right)^{6x}$.

10.21 $\lim\limits_{x\to -1}\dfrac{7x^2+4x-3}{2x^2+3x+1}$. $\lim\limits_{x\to -6}\dfrac{\sqrt{2x+13}-\sqrt{7+x}}{x^2+5x-6}$.

$\lim\limits_{x\to 0}\dfrac{x\cdot tg4x}{arctg2x}$. $\lim\limits_{x\to 1}\left(\dfrac{3x-1}{x+1}\right)^{\frac{1}{\sqrt{x}-1}}$.

10.22. $\lim\limits_{x\to 1}\dfrac{4x^4-5x^2+1}{x^2-1}$. $\qquad \lim\limits_{x\to 16}\dfrac{\sqrt[4]{x}-2}{4-\sqrt{x}}$.

$\lim\limits_{x\to \pi}\dfrac{\pi^2-x^2}{1-\cos^2 x}$. $\qquad \lim\limits_{x\to -1}(2x+3)^{\frac{3x}{x+1}}$.

10.23. $\lim\limits_{x\to -3}\dfrac{x^3+27}{2x^2+5x-3}$. $\qquad \lim\limits_{x\to -1}\dfrac{3x^2+4x+1}{\sqrt{8-x}-\sqrt{4-5x}}$.

$\lim\limits_{x\to \frac{\pi}{4}}\dfrac{\cos x-\sin x}{1-ctgx}$. $\qquad \lim\limits_{x\to 2}(3x-5)^{\frac{x^2}{x^2-4}}$.

10.24. $\lim\limits_{x\to 1}\dfrac{x^3-3x+2}{x^2-4x+3}$. $\qquad \lim\limits_{x\to -2}\dfrac{x^2+x-2}{\sqrt{3x+11}-\sqrt{1-2x}}$.

$\lim\limits_{x\to 0}\dfrac{\sin 2x}{\sin 3x-\sin x}$. $\qquad \lim\limits_{x\to \infty}(3x+1)[\ln(2x-1)-\ln(2x+1)]$.

10.25. $\lim\limits_{x\to -1}\dfrac{x^4-1}{x^4-x^2+x+1}$. $\qquad \lim\limits_{x\to -3}\dfrac{2x^2+3x-9}{\sqrt{x+10}-\sqrt{4-x}}$.

$\lim\limits_{x\to 0}\dfrac{\cos x-\cos^5 x}{x\cdot\sin 2x}$. $\qquad \lim\limits_{x\to -2}(4x+9)^{\frac{5x}{2+x}}$.

11-masala. Funksiyani uzluksizlikka tekshiring va grafigini chizing.

11.1. $f(x)=\begin{cases}\sqrt{1-x}, & x\le 0,\\ 0, & 0<x\le 2,\\ x-2, & x>2.\end{cases}$ $\qquad$ **11.2.** $f(x)=\begin{cases}x-3, & x<0,\\ x+1, & 0\le x\le 3,\\ 7-x, & x>3.\end{cases}$

11.3. $f(x)=\begin{cases}x+4, & x<-1,\\ x^2+2, & -1\le x<1,\\ 3x, & x\ge 1.\end{cases}$ $\qquad$ **11.4.** $f(x)=\begin{cases}x^2, & x\le 0,\\ 0, & 0<x\le 2,\\ 2-x, & x>2.\end{cases}$

11.5. $f(x)=\begin{cases}-2(x+1), & x\le -1,\\ x^2, & -1<x\le 3,\\ x-1, & x>3.\end{cases}$ $\qquad$ **11.6.** $f(x)=\begin{cases}-x, & x\le 0,\\ x^3, & 0<x\le 1,\\ x+1, & x>1.\end{cases}$

11.7. $f(x)=\begin{cases}x, & x\le -2,\\ -x+1, & -2<x\le 1,\\ x^2-1, & x>1.\end{cases}$ $\qquad$ **11.8.** $f(x)=\begin{cases}1, & x<0,\\ \cos x, & 0\le x\le \pi,\\ 1-x, & x>\pi.\end{cases}$

11.9. $f(x)=\begin{cases}x+3, & x\le 0,\\ -x^2+4, & 0<x<2,\\ x-2, & x\ge 2.\end{cases}$ $\qquad$ **11.10.** $f(x)=\begin{cases}x-1, & x\le 0,\\ \sin x, & 0<x<\pi,\\ 3, & x\ge \pi.\end{cases}$

11.11. $f(x)=\begin{cases} -x, & x\le 0, \\ -(x-1)^2, & 0<x<2, \\ x-2, & x\ge 2. \end{cases}$ **11.12.** $f(x)=\begin{cases} x^2+1, & x\le 1, \\ 2x, & 1<x<3, \\ x+3, & x\ge 3. \end{cases}$

11.13. $f(x)=\begin{cases} x^3, & x\le -1, \\ x-1, & -1<x\le 3, \\ -x+5, & x>3. \end{cases}$ **11.14.** $f(x)=\begin{cases} \cos x, & x\le \dfrac{\pi}{2}, \\ 0, & \dfrac{\pi}{2}<x<\pi, \\ 2-x, & x\ge \pi. \end{cases}$

11.15. $f(x)=\begin{cases} x+2, & x\le -1, \\ x^2+1, & -1<x\le 1, \\ -x+3, & x>1. \end{cases}$ **11.16.** $f(x)=\begin{cases} x^3, & x\le 0, \\ (x-1)^2, & 0<x\le 3, \\ x+1, & x>3. \end{cases}$

11.17. $f(x)=\begin{cases} -2(x+1), & x\le -1, \\ x^2, & -1<x\le 3, \\ x^2-1, & x>3. \end{cases}$ **11.18.** $f(x)=\begin{cases} -x, & x\le 0, \\ x^2, & 0<x\le 1, \\ x+1, & x>1. \end{cases}$

11.19. $f(x)=\begin{cases} x, & x\le -2, \\ -x+1, & -2<x\le 1, \\ x^2-1, & x>1. \end{cases}$ **11.20.** $f(x)=\begin{cases} 1, & x<0, \\ \sin x, & 0\le x\le \pi, \\ 1+x, & x>\pi. \end{cases}$

11.21. $f(x)=\begin{cases} 3x+1, & x\le 0, \\ -x^2+4, & 0<x<2, \\ x-2, & x\ge 2. \end{cases}$ **11.22.** $f(x)=\begin{cases} x+1, & x\le 0, \\ \cos x, & 0<x<\pi, \\ 4, & x\ge \pi. \end{cases}$

11.23. $f(x)=\begin{cases} 1-2x, & x<-1, \\ x^2+2, & -1\le x<1, \\ 4x, & x\ge 1. \end{cases}$ **11.24.** $f(x)=\begin{cases} 3x, & x\le 0, \\ 0, & 0<x\le 2, \\ x^2-2x-x, & x>2. \end{cases}$

11.25. $f(x)=\begin{cases} x^2, & x\le -1, \\ x-1, & -1<x\le 3, \\ -2x+8, & x>3. \end{cases}$

12-masala. Funksiyani berilgan nuqtalarda uzluksizlikka tekshiring

12.1. $f(x)=\dfrac{x-5}{x-2};\quad x_1=3,\quad x_2=2.$ **12.2.** $f(x)=2^{\frac{1}{x-4}};\quad x_1=4,\quad x_2=5.$

12.3. $f(x)=\dfrac{4x}{x+5}$; $x_1=3,\quad x_2=-5.$

12.4. $f(x)=3^{\frac{2}{x+2}}$; $x_1=-1,\quad x_2=-2.$

12.5. $f(x)=\dfrac{2x}{x^2-1}$; $x_1=1,\quad x_2=2.$

12.6. $f(x)=7^{\frac{4}{x-3}}$; $x_1=2,\quad x_2=4.$

12.7. $f(x)=4^{\frac{x}{1-x}}$; $x_1=1,\quad x_2=2.$

12.8. $f(x)=\dfrac{3x}{4-x^2}$; $x_1=2,\quad x_2=3.$

12.9. $f(x)=5^{\frac{1}{x-3}}$; $x_1=3,\quad x_2=4.$

12.10. $f(x)=6^{\frac{1}{3+x}}$; $x_1=-2,\quad x_2=-3.$

12.11. $f(x)=\dfrac{x+5}{x-2}$; $x_1=2,\quad x_2=3.$

12.12. $f(x)=8^{\frac{4}{x+2}}$; $x_1=-3,\quad x_2=-2.$

12.13. $f(x)=\dfrac{x}{x^3+8}$; $x_1=-2,\quad x_2=-1.$

12.14. $f(x)=5^{\frac{3}{x+4}}$; $x_1=-4,\quad x_2=-3.$

12.15. $f(x)=\dfrac{3x}{x-5}$; $x_1=3,\quad x_2=5.$

12.16. $f(x)=6^{\frac{2}{x+1}}$; $x_1=-1,\quad x_2=-2.$

12.17. $f(x)=\dfrac{2x}{x^2-4}$; $x_1=1,\quad x_2=2.$

12.18. $f(x)=5^{\frac{4}{x-3}}$; $x_1=2,\quad x_2=5.$

12.19. $f(x)=4^{\frac{x}{1-x}}$; $x_1=1,\quad x_2=2.$

12.20. $f(x)=\dfrac{3x}{4-x^2}$; $x_1=2,\quad x_2=3.$

12.21. $f(x)=4^{\frac{1}{x-4}}$; $x_1=3,\quad x_2=4.$

12.22. $f(x)=5^{\frac{1}{5+x}}$; $x_1=-5,\quad x_2=-3.$

12.23. $f(x)=\dfrac{x+5}{x^2-1}$; $x_1=1,\quad x_2=3.$

12.24. $f(x)=7^{\frac{4}{x+2}}$; $x_1=-4,\quad x_2=-2.$

12.25. $f(x)=\dfrac{x^2-1}{x+2}$; $x_1=-3,\quad x_2=-2.$